Measurement of the two-photon exchange contribution in elastic $ep$ scattering at VEPP–3


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Proton electromagnetic form factors

The proton’s electric $G_E(Q^2)$ and magnetic $G_M(Q^2)$ form factors are very important characteristics of this particle. They describe the distribution of charge and magnetism of the proton as functions of squared four-momentum transfer $Q^2$.

Differential cross section of the elastic $ep$ scattering in one-photon exchange approximation gives by the Rosenbluth formula:

$$\frac{d\sigma}{d\Omega} = \frac{\tau}{\varepsilon(1+\tau)} \left[ G_M^2 + \frac{\varepsilon}{\tau} G_E^2 \right] \frac{d\sigma_0}{d\Omega},$$

where $\tau = Q^2/(4M^2)$, $\varepsilon = \left[ 1 + 2(1+\tau)\tan^2(\theta/2) \right]^{-1}$ is virtual photon polarization, $M$ is the proton mass, $\theta$ is electron scattering angle. $d\sigma_0/d\Omega$ is the Mott differential cross section including the proton recoil.

Until the 1990’s the experimental method to determine $G_E(Q^2)$ and $G_M(Q^2)$ was based on the measurements of the unpolarized elastic $ep$ scattering cross sections at fixed four-momentum transfer, but with different electron scattering angles and incident beam energies. Such a procedure is called Rosenbluth separation or Rosenbluth technique.

Unpolarized and polarized elastic $ep$ scattering

The first such measurements were reported in 1956 by Robert Hofstadter.

It was found that both the electric and magnetic form factors are well described by the so-called dipole formula

$$G_E(Q^2) = \left(1 + \frac{Q^2}{0.71 \text{ GeV}^2}\right)^{-2}, \quad G_M(Q^2) = \mu G_E(Q^2),$$

implying a ratio of $\mu G_E/G_M \approx 1$, where $\mu \approx 2.79$ is the proton magnetic moment.

The form factors also can be measured in polarization transfer experiments using $(\vec{e}, e' \vec{p})$ scattering.

In such a case the ratio $P_t/P_l$ of transverse $P_t$ and longitudinal $P_l$ recoil proton polarizations is directly proportional to $G_E/G_M$:

$$\frac{G_E}{G_M} = -\frac{P_t}{P_l} \frac{E + E'}{2M} \tan \frac{\theta}{2}$$

A series of precise measurements of the ratio $G_E/G_M$ for a wide range of $Q^2$ was carried out recently at Jefferson Lab (USA).
There is a dramatic difference between polarized and unpolarized data!
Two-photon exchange contribution?

This discrepancy has been explained as the effect of two-photon exchange (TPE) beyond the usual one-photon exchange approximation in the calculation of the elastic electron-proton scattering cross section.

Complications arising in the calculation of the TPE corrections are connected with difficulties in accounting for proton excitations in the intermediate state.

Fortunately, the contribution of two-photon exchange can be measured directly. This is possible due to the fact that the TPE corrections have opposite signs for $e^+p$ and $e^-p$ scattering cross sections, producing a measurable charge asymmetry

$$R = \frac{\sigma(e^+p)}{\sigma(e^-p)},$$

where $\sigma(e^+)$ and $\sigma(e^-)$ denote elastic cross sections of positron-proton and electron-proton scattering, respectively.
A measurement of the ratio $R = \sigma(e^+ p)/\sigma(e^- p)$ has been performed recently at the VEPP–3 storage ring at the energy of electron/positron beams of 1.6 GeV and at lepton scattering angles around $10^\circ$, $18^\circ$ and $64^\circ$. The smallest angle region is used for luminosity monitoring.

<table>
<thead>
<tr>
<th>$\theta_\ell$ ($^\circ$)</th>
<th>$Q^2$ (GeV$^2$)</th>
<th>$\varepsilon$</th>
<th>$\Delta R/R$ (% stat.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SA</td>
<td>$8.4 \div 12.9$</td>
<td>$0.05 \div 0.13$</td>
<td>$0.99 \div 0.97$</td>
</tr>
<tr>
<td>MA</td>
<td>$15.5 \div 22.4$</td>
<td>$0.18 \div 0.34$</td>
<td>$0.96 \div 0.92$</td>
</tr>
<tr>
<td>LA</td>
<td>$57.5 \div 71.0$</td>
<td>$1.32 \div 1.61$</td>
<td>$0.55 \div 0.40$</td>
</tr>
</tbody>
</table>

Internal hydrogen gas target was used (thickness: $5 \cdot 10^{14}$ at./cm$^2$). Data collection with electron and positron beams was alternated regularly, so that one cycle with two beams required 1.5 hours. Experiment duration was about 1500 hours with a mean luminosity of $5 \cdot 10^{31}$ s$^{-1}$ · cm$^{-2}$.

Also two similar experiments have been proposed to study the $e^+ p$ and $e^- p$ cross sections ratio: one at DORIS storage ring (OLYMPUS collaboration), and another at Jefferson Lab (CLAS collaboration) using a secondary electron/positron beam from a pair production target.

- Proposal for the experiment at VEPP–3 (2004): nucl-ex/0408020
- OLYMPUS collaboration: http://web.mit.edu/OLYMPUS/
- Proposal for the experiment at CLAS (Jefferson Lab): PR-07-005.pdf
The VEPP–3 electron-positron storage ring

1 — Girokon (430 MHz)
2 — Linac (50 MeV)
3 — Electron to positron converter
4 — Synchrotron B–4 (350 MeV)

Perimeter: 74.4 m
Injection energy: 350 MeV
Maximal energy: 2000 MeV
Maximal $e^+$ current: 50 mA
Schematic side view of the particle detection system

- Plastic scintillators
- Drift chambers
- Proportional chambers
- e^+/e^- beam \( E = 1.6 \text{ GeV} \)
- Storage cell (H_2 target)
- 0.5 m

Sandwiches at small angle

Aperture counters

Measurement of the two-photon...
The detector and the target installed at VEPP-3
Internal target — storage cell cooled by Cold Head

Cold Head
Cryomech GB37
The main sources of systematic uncertainties

- **Different beam positions for electrons and positrons**
  This effect can be suppressed in first order by averaging count rates of up-arm and down-arm. We had three sources of information on the $e^−/e^+$ beam positions: VEPP–3 beam position monitors, moveable beam scrapers and vertex position reconstruction using coordinate system of the detector.

- **Unequal beam energy for electrons and positrons**
  Electron/positron beams energies were measured regularly and with good accuracy (better than 200 keV) during data taking by Compton backscattering setup.

- **Time instability of detection efficiency**

- **Drift of the target thickness in time**
  Data collection with electron and positron beams was alternated regularly. This allows us to suppress effects of drift in time of the target thickness and detection efficiency.

- **Uncertainty related to radiative corrections**

  Full systematic uncertainty is estimated now as $3 \cdot 10^{-3}$ (0.3%).
The figure shows the beam current (green for $e^-$, red for $e^+$) and lifetime (black). Alternation of $e^-$ and $e^+$ beams allows to suppress effects of drift in time the target thickness and detection efficiency.
Beam energy measurement by Compton backscattering

Already existing system created earlier for the VEPP–4 collider has been used.

Photons from a CO$_2$ laser are scattered in a head-on collision with the stored beam. From the spectrum of the backscattered photons that are detected by an energy-calibrated high purity Ge detector the beam energy can be determined.

\[
E = \frac{\omega_{\text{max}}}{2} \left( 1 + \sqrt{1 + \frac{m_e^2}{\omega_0 \omega_{\text{max}}}} \right),
\]

where $\omega_{\text{max}}$ — maximal energy of backscattering photons (the edge of spectrum), $\omega_0$ — energy of laser photons, $m_e$ — electron mass.
The beam energy was measured by Compton backscattering with an accuracy of less than 200 keV.
Why is it important to take into account the RC?

The diagrams of ep scattering in the $1\gamma$ and $2\gamma$ approximations.

Virtual photon corrections, which don’t depend on the detector geometry.

Corrections related to the bremsstrahlung of the first order. Their contribution is determined by the detector geometry!

The experimentally measured ratio $R = \sigma(e^+ p)/\sigma(e^- p)$:

$$R \approx \frac{e^4|\mathcal{M}_{\text{Born}}|^2 + 2e^6\mathcal{M}_{\text{Born}}\text{Re}(\mathcal{M}^*_{2\gamma}) + e^6|\mathcal{M}_{ei} + \mathcal{M}_{ef} + \mathcal{M}_{pi} + \mathcal{M}_{pf}|^2 + \ldots}{e^4|\mathcal{M}_{\text{Born}}|^2 - 2e^6\mathcal{M}_{\text{Born}}\text{Re}(\mathcal{M}^*_{2\gamma}) + e^6| - \mathcal{M}_{ei} - \mathcal{M}_{ef} + \mathcal{M}_{pi} + \mathcal{M}_{pf}|^2 + \ldots}$$
Typically, the following formula is used to account for the RC:

\[
\frac{d\sigma}{d\Omega_e} (E_\gamma < \Delta E) = \frac{d\sigma_{\text{Born}}}{d\Omega_e} \left[1 - \delta_1(\Delta E) - \delta_2\right],
\]

where \(\Delta E\) is parameter that characterizes the energy resolution of the detector; \(\delta_1(\Delta E)\) is the correction related to the bremsstrahlung; \(\delta_2\) is the correction related to virtual photons.

- The soft photon approximation is commonly used in the calculations of the term \(\delta_1(\Delta E)\).
- Infrared divergences in the diagrams of bremsstrahlung and two-photon exchange diagrams cancel each other.
- Exponentiation is used to account for the multiphoton emission:

\[
\frac{d\sigma}{d\Omega_e} (E_\gamma < \Delta E) = \frac{d\sigma_{\text{Born}}}{d\Omega_e} e^{-\delta_1(\Delta E)}(1 - \delta_2).
\]

Simple estimate based on the work by Maximon and Tjon can be made. This estimate gives $R \approx 1.03$ for the LA region (see the blue curve on the figure).

Radiative corrections in our experiment

- Calculation by V. S. Fadin and A. L. Feldman is used to account for the first order bremsstrahlung. FeynCalc package is used in this calculation.
- Calculation by V. S. Fadin and R. E. Gerasimov is used to estimate the contribution of the first order bremsstrahlung with the delta-isobar \( \Delta(1232) \) excitation. FeynCalc package is used again. As far as we know, this is the first calculation of the process.

\[ \Delta \quad \Delta \]

- Soft photon approximation is not used in these calculations (there is no limit on the photon energy).
- Based on the calculations, we have created an event generator called ESEPP (Elastic Scattering of Electrons and Positrons by Protons). ESEPP generates events \((ep \rightarrow ep\) and \(ep \rightarrow ep\gamma\)), which are then used to simulate the experiment in the Geant4 toolkit.
- Geant4 simulation allows us to accurately account for the detector geometry and kinematic cuts used in event selection (angular correlations for the detected lepton and proton; cut on the lepton energy).
Three calculations for the radiative corrections

LA region \((\theta_\ell = 57^\circ \div 71^\circ)\), \(\Delta E = 15\%\).
Preliminary results of the experiment

Middle angle ($\varepsilon = 0.95, Q^2 = 0.23$ GeV$^2$): $R = 1.0024 \pm 0.0009 \pm 0.003$,

Large angle ($\varepsilon = 0.5, Q^2 = 1.43$ GeV$^2$): $R = 1.018 \pm 0.011 \pm 0.003$.

Our preliminary results and the previous measurements (for $Q^2 < 2$ GeV$^2$):

- Yount (1962);
- Browman (1965), run 1;
- Browman (1965), run 2;
- Anderson (1966);
- Bartel (1967);
- Bouquet (1968);
- Anderson (1968);
- Mar (1968);
- this experiment.
The next phase of the experiment

The measurement will be continued at other kinematics: 
\( \varepsilon = 0.42, \ Q^2 = 0.82 \text{ GeV}^2 \) & \( \varepsilon = 0.29, \ Q^2 = 0.96 \text{ GeV}^2 \).

The figure shows projected statistical accuracy (blue squares) and our preliminary results (black circles). The lines represent the corresponding results of the theoretical prediction by Blunden, et al.

Conclusion & Outlook

- The first precision measurement of the ratio $R = \sigma(e^+ p)/\sigma(e^- p)$ has been performed. Preliminary results of the experiment have been presented.
- Procedure of account of the radiative corrections has been developed (ESEPP event generator + Geant4 simulation of the detector).
- Exact calculation by V. S. Fadin and A. L. Feldman has been used to account for the first order bremsstrahlung beyond the soft photon approximation.
- Calculation by V. S. Fadin and R. E. Gerasimov has been used to estimate the contribution of the first order bremsstrahlung with the delta-isobar $\Delta(1232)$ excitation. As far as we know, this is the first calculation of the process.
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Bremsstrahlung in the soft-photon approximation (SPA)
Selection of the elastic $ep$ scattering events
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Kinematics of the reaction:

\[ \ell + p = \ell' + p' + k, \]

where \( \ell, \ell' \) — initial and final lepton four-momenta; \( p, p' \) — initial and final proton four-momenta; \( k \) — photon four-momentum.

Formulas simplify greatly in the limit that the photon energy \( E_\gamma \) is much less than the momenta of the initial lepton and final state fermions. This limit is referred to as the soft-photon approximation (SPA). In this case, the differential cross section for single-photon bremsstrahlung is given by

\[
\frac{d\sigma}{d\Omega_\ell d\Omega_\gamma dE_\gamma} = \frac{-\alpha E_\gamma}{4\pi^2} \left[ \pm \frac{\ell'}{k \cdot \ell'} - \frac{p'}{k \cdot p'} \mp \frac{\ell}{k \cdot \ell} + \frac{p}{k \cdot p} \right]^2 \frac{d\sigma_0}{d\Omega_\ell},
\]

where \( d\sigma_0/d\Omega_\ell \) is the one-photon exchange electron-proton cross section (given by Rosenbluth formula). The first and third terms in brackets have different signs in the case of \( e^- p \) and \( e^+ p \) scattering.

Selection of the elastic $ep$ scattering events

1. Correlation between polar angles ($\theta_\ell$ vs. $\theta_p$)
2. Correlation between azimuthal angles ($\phi_\ell$ vs. $\phi_p$)
3. Correlation between lepton scattering angle and proton energy ($\theta_\ell$ vs. $E_p$)
4. Correlation between lepton scattering angle and electron energy ($\theta_\ell$ vs. $E_\ell$)
5. $\Delta E - E$ analysis
6. Time-of-flight analysis for low energy protons