Threshold pion production in DIS

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based on the collaboration with V. Braun, A. Lenz and A. Peters

International Workshop on e+e- collisions from Phi to Psi

Outline

Soft Pion Limit

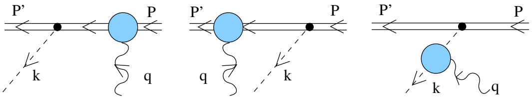
pion electroproduction cross section close to threshold $W \to W_{\rm th}$

$$\begin{aligned} e(l) + p(P) &\to e(l') + \pi^+(k) + n(P')\,, \\ e(l) + p(P) &\to e(l') + \pi^0(k) + p(P')\,. \end{aligned}$$

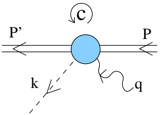
$$\begin{aligned} W^2 &= (P' + k)^2 \\ W_{\rm th} &= m_N + m_\pi \\ Q^2 &= -q^2 = -(\ell - \ell')^2 \end{aligned}$$

can be calculated in the chiral limit $m_\pi \to 0$, $|\vec{k}| = \mathcal{O}(m_\pi)$, $Q = \mathcal{O}(m_\pi)$ in terms of

- **Pion emission from external legs**



- **Chiral Rotation**



$$\langle \pi^a N | j_\mu^{\rm em} | N \rangle \sim \frac{i}{f_\pi} \langle N | [j_\mu^{\rm em}, Q_5^a] | N \rangle$$

Kroll, Ruderman '54

Generalized Form Factors

at the threshold

$$\langle \pi N | j_\mu^{\rm em} | p \rangle = -\frac{i}{f_\pi} \bar{N}(P') \gamma_5 \left\{ \left(\gamma_\mu q^2 - q_\mu \not{q} \right) \frac{1}{m_N^2} G_1^{\pi N}(Q^2) - \frac{i \sigma_{\mu\nu} q^\nu}{2 m_N} G_2^{\pi N}(Q^2) \right\} N(P)$$

related to S-wave multipoles in the PWA

$$E_{0+}^{\pi N} = \frac{\sqrt{4\pi\alpha_{\rm em}}}{8\pi} \frac{Q^2 \sqrt{Q^2 + 4 m_N^2}}{m_N^3 f_\pi} G_1^{\pi N}$$

$$L_{0+}^{\pi N} = \frac{\sqrt{4\pi\alpha_{\rm em}}}{32\pi} \frac{Q^2 \sqrt{Q^2 + 4 m_N^2}}{m_N^3 f_\pi} G_2^{\pi N}$$

e.g. the differential cross section at threshold is given by

$$\left. \frac{d\sigma_{\gamma^*}}{d\Omega_\pi} \right|_{\rm th} = \frac{2|\vec{k}_f| W}{W^2 - m_N^2} \left[(E_{0+}^{\pi N})^2 + \epsilon \frac{Q^2}{(\omega_\gamma^{\rm th})^2} (L_{0+}^{\pi N})^2 \right]$$

small Q^2

PCAC + current algebra:

Vainshtein, Zakharov, NPB36(1972)589
Scherer, Koch, NPA534(1991)461

$$\frac{Q^2}{m_N^2} G_1^{\pi^0 p} = \frac{g_A}{2} \frac{Q^2}{(Q^2 + 2m_N^2)} G_M^p \,, \qquad G_2^{\pi^0 p} = \frac{2 g_A m_N^2}{(Q^2 + 2m_N^2)} G_E^p \,,$$

$$\frac{Q^2}{m_N^2} G_1^{\pi^+ n} = \frac{g_A}{\sqrt{2}} \frac{Q^2}{(Q^2 + 2m_N^2)} G_M^n + \frac{1}{\sqrt{2}} G_A \,, \qquad G_2^{\pi^+ n} = \frac{2\sqrt{2} g_A m_N^2}{(Q^2 + 2m_N^2)} G_E^n \,,$$

- **Threshold photoproduction of π^0 is suppressed compared to π^+**
- **The π^0/π^+–ratio is rapidly increasing with Q^2**

How far in Q^2 can one go? What happens for $Q^2 \gg m_N^2$?

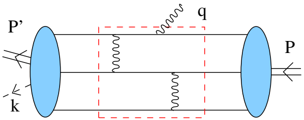
◇ **Do not address the $\mathcal{O}(m_\pi)$ corrections in this study**

very large Q^2

- **At large momentum transfers, the pion cannot be soft to the initial and the final state nucleons simultaneously**
- **The corresponding kinematic condition is** $Q^2 > \Lambda^3/m_\pi, \quad \Lambda \sim 1\text{GeV}$
- **Asymptotic limit $Q^2 \to \infty$ does not commute with the chiral limit $m_\pi \to 0$**

QCD factorization for $Q^2 \gg \Lambda^3/m_\pi$

Pobylitsa, Polyakov, Strikman, PRL87(2001)022001:



◇ **Probably unrealistic at reachable momentum transfers**

Pion–Nucleon Distribution Amplitudes

$$|p\uparrow\rangle = \frac{\phi_s(x)}{\sqrt{6}}|2u_\uparrow d_\downarrow u_\uparrow - u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle + \frac{\phi_a(x)}{\sqrt{2}}|u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle$$

$$|p\uparrow\pi^0\rangle = \frac{\phi_s(x)}{2\sqrt{6}f_\pi}|6u_\uparrow d_\downarrow u_\uparrow + u_\uparrow u_\downarrow d_\uparrow + d_\uparrow u_\downarrow u_\uparrow\rangle - \frac{\phi_a(x)}{2\sqrt{2}f_\pi}|u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle$$

$$|n\uparrow\pi^+\rangle = \frac{\phi_s(x)}{\sqrt{12}f_\pi}|2u_\uparrow d_\downarrow u_\uparrow - 3u_\uparrow u_\downarrow d_\uparrow - 3d_\uparrow u_\downarrow u_\uparrow\rangle - \frac{\phi_a(x)}{2f_\pi}|u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle$$

Pobylitsa, Polyakov, Strikman; PRL87(2001)022001

Equivalent representation

$$4\langle 0|\epsilon^{ijk}u^i_\alpha(a_1z)u^j_\beta(a_2z)d^k_\gamma(a_3z)|N(P)\pi\rangle_{\text{twist}-3} =$$

$$= (\gamma_5)_{\gamma\delta}\frac{-i}{f_\pi}\left[V_1^{\pi N}(\not{p}C)_{\alpha\beta}\left(\gamma_5 N^+\right)_\gamma + A_1^{\pi N}(\not{p}\gamma_5 C)_{\alpha\beta}N^+_\gamma + T_1^{\pi N}(i\sigma_{\perp p}C)_{\alpha\beta}\left(\gamma^\perp\gamma_5 N^+\right)_\gamma\right]$$

$$V_1^{n\pi^+}(1,2,3) = \frac{1}{\sqrt{2}}\left\{V_1^n(1,3,2) + V_1^n(1,2,3) + V_1^n(2,3,1) + A_1^n(1,3,2) + A_1^n(2,3,1)\right\},$$

$$A_1^{n\pi^+}(1,2,3) = -\frac{1}{\sqrt{2}}\left\{V_1^n(3,2,1) - V_1^n(1,3,2) + A_1^n(2,1,3) + A_1^n(2,3,1) + A_1^n(3,1,2)\right\},$$

$$T_1^{n\pi^+}(1,2,3) = \frac{1}{2\sqrt{2}}\left\{A_1^n(2,3,1) + A_1^n(1,3,2) - V_1^n(2,3,1) - V_1^n(1,3,2)\right\}.$$

Braun, Ivanov, Lenz, A.Peters; PRD75(2007)014021

Extended to twist-4,5,6

Light–Cone Sum Rules: General Strategy

Balitsky, V.B., Kolesnichenko '86–'88

- **consider**

$$T_\nu^{\pi N}(P,q) = i\int d^4x\, e^{iqx}\langle 0|T\{\eta_p(0) j_\nu^{\rm em}(x)|N(P)\pi^a(k)\rangle$$

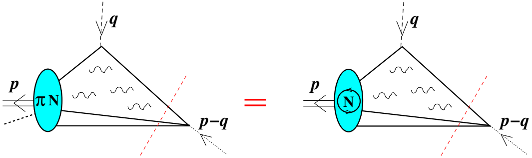
$$\eta_p(x) = \epsilon^{ijk}\left[u^i(x)C\gamma_\mu u^j(x)\right]\gamma_5\gamma^\mu d^k(x)\,, \qquad \langle 0|\eta_p|N(P)\rangle = \lambda_p m_N N(P)$$

- **take $(P-q)^2 \sim -1$ GeV^2 and make a matching between**

(a) **The Operator Product Expansion in terms of pion-nucleon DAs**

$$\langle 0|T\{\eta_p(0) j_\nu^{\rm em}(x)|N(P)\pi^a(k)\rangle = \sum_{\rm twist} C_\nu(x^2, px)\otimes\langle 0|q(x_1)q(x_2)q(x_3)|N(P)\pi^a(k)\rangle$$

(b) **The dispersion representation in terms of hadronic states**



- ◇ **Borel transformation to improve convergence**
- ◇ **Here, pion in initial state (convenient)**

Light–Cone Sum Rules: Pion–Nucleon Intermediate States

- **New: Semidisconnected pion-nucleon contributions in the intermediate state**

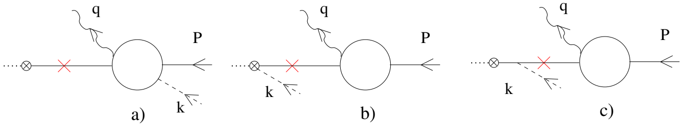


Figure: Schematic structure of the pole terms in the correlation function

b) and c) correspond to πN coupling to the Ioffe current

$$\langle 0|\,\eta_p(0)\,|N(P'-k)\pi(k)\rangle \;=\; \frac{i\lambda_1^p m_N}{2f_\pi}\left[1-\frac{g_A}{P'^2-m_N^2}(\not P'-\not k+m_N)\,\not k\right]\gamma_5 N(P'-k)\,.$$

In the threshold kinematics, with $\delta = m_\pi/m_N$

$$T_\nu^{\pi^0 p}(P,q) = \frac{i\lambda_1^p m_N}{f_\pi}\left\{\frac{((1+\delta)\not P-\not q+m_N)\gamma_5}{m_N^2-P'^2}\left[(\gamma_\nu q^2-q_\nu\not q)\frac{G_1^{\pi^0 p}}{m_N^2}-\frac{i\sigma_{\nu\mu}q^\mu}{2m_N}G_2^{\pi^0 p}\right]\right.$$
$$+\frac{1}{2}\frac{(1+\delta)\gamma_5(\not P-\not q+m_N)}{[m_N^2(1+\delta)^2+\delta Q^2]-P'^2}\left[\gamma_\nu F_1^p-\frac{i\sigma_{\nu\mu}q^\mu}{2m_N}F_2^p\right]$$
$$\left.-\frac{(1+\delta)g_A(\not P-\not q+m_N)\gamma_5}{[m_N^2(1+\delta)^2+\delta Q^2]-P'^2}\left[(\gamma_\nu q^2-q_\nu\not q)G_M^p-\frac{i\sigma_{\nu\mu}q^\mu}{2m_N}4m_N^2G_E^p\right]\right\}N(P)$$

Light–Cone Sum Rules: Pion–Nucleon Intermediate States — *cont.*

- **The semidisconnected πN contributions can be included in the continuum if**

$$m_\pi Q^2 > m_N(s_0 - m_N^2) \quad \Rightarrow \quad Q^2 > 7\,\mathrm{GeV}^2 \qquad [\sim \Lambda^3/m_\pi]$$

- **Otherwise they have to be taken into account explicitly**

$$\frac{Q^2}{m_N^2} G_1^{\pi^0 p} = \frac{e^{m_N^2/M^2}}{2\lambda_1^p} \mathbb{B}_{P'2}[\mathcal{A}^{\pi^0 p}](M^2, Q^2) - \frac{1}{2} e^{-\delta(2m_N^2+Q^2)/M^2} \left[F_1^p(Q^2) - \frac{g_A Q^2}{Q^2 + 2m_N^2} G_M^p(Q^2) \right]$$

$$G_2^{\pi^0 p} = -\frac{e^{m_N^2/M^2}}{\lambda_1^p} \mathbb{B}_{P'2}[\mathcal{B}^{\pi^0 p}](M^2, Q^2) + e^{-\delta(2m_N^2+Q^2)/M^2} \left[\frac{1}{2} F_2^p(Q^2) + \frac{2g_A m_N^2}{Q^2 + 2m_N^2} G_E^p(Q^2) \right]$$

$$\frac{Q^2}{m_N^2} G_1^{\pi^+ n} = \frac{e^{m_N^2/M^2}}{2\lambda_1^p} \mathbb{B}_{P'2}[\mathcal{A}^{\pi^+ n}](M^2, Q^2) - \frac{1}{\sqrt{2}} e^{-\delta(2m_N^2+Q^2)/M^2} \left[F_1^n(Q^2) - \frac{g_A Q^2}{Q^2 + 2m_N^2} G_M^n(Q^2) \right]$$

$$G_2^{\pi^+ n} = -\frac{e^{m_N^2/M^2}}{\lambda_1^p} \mathbb{B}_{P'2}[\mathcal{B}^{\pi^+ n}](M^2, Q^2) + e^{-\delta(2m_N^2+Q^2)/M^2} \left[\frac{1}{\sqrt{2}} F_2^n(Q^2) + \frac{2\sqrt{2} g_A m_N^2}{Q^2 + 2m_N^2} G_E^n(Q^2) \right]$$

where $\mathcal{A}(P'^2, Q^2)$ and $\mathcal{A}(P'^2, Q^2)$ are the invariant functions defined as

$$z^\nu \Lambda^+ T_\nu^{\pi N}(P, q) = \frac{i}{f_\pi} (pz + kz)\gamma_5 \left\{ m_N \mathcal{A}(P'^2, Q^2) + \not{q}_\perp \mathcal{B}(P'^2, Q^2) \right\} N^+(P)$$

Light–Cone Sum Rules: Results

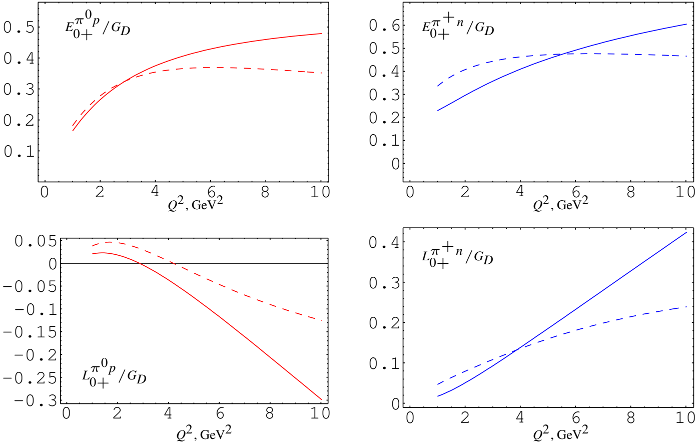


Figure: The LCSR prediction for E_{0+} and L_{0+} at threshold.

- **normalized to the dipole formula** $G_D = 1/(1 + Q^2/0.71)^2$

Away from the threshold ...

for $W - W_{\rm th} \ll m_\pi$ **accept**

$$\langle N(P')\pi(k)|j_\mu^{em}(0)|p(P)\rangle =$$

$$= \quad -\frac{i}{f_\pi}\bar{N}(P')\gamma_5\left\{\left(\gamma_\mu q^2 - q_\mu \not{q}\right)\frac{1}{m_N^2}G_1^{\pi N}(Q^2) - \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}G_2^{\pi N}(Q^2)\right\}N(P)$$

$$+\frac{ic_\pi g_A}{2f_\pi\,[(P'+k)^2)-m_N^2]}\bar{N}(P')\not{k}\gamma_5(\not{P}'+m_N)\left\{F_1^p(Q^2)\left(\gamma_\mu - \frac{q_\mu \not{q}}{q^2}\right) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}F_2^p(Q^2)\right\}N(P)$$

- **S–wave: generalized form factors from LCSR**
- **P–wave: pion emission from the final state nucleon; exact in chiral limit**
- **Eventually can take into account the final state interactions**

$$G_1^{\pi N}(Q^2) \to G_1^{\pi N}(Q^2, W) \equiv G_1^{\pi N}(Q^2)[1 + i\,t_{\pi N}]$$

Structure Functions at $x_B \to 1$

$$F_1(W,Q^2) = \frac{\beta(W)}{(4\pi f_\pi)^2} \sum_{\pi^0,\pi^+} \left\{ \frac{Q^2 + 4m_N^2}{2m_N^4} |Q^2 G_1^{\pi N}|^2 + \frac{c_\pi^2 g_A^2 W^2 \beta^2(W)}{8(W^2 - m_N^2)^2} Q^2 m_N^2 G_M^2 \right\}$$

$$F_2(W,Q^2) = \frac{\beta(W)}{(4\pi f_\pi)^2} \sum_{\pi^0,\pi^+} \left\{ \frac{Q^2}{m_N^4} \left(|Q^2 G_1^{\pi N}|^2 + \frac{m_N^2}{4} Q^2 |G_2^{\pi N}|^2 \right) + \frac{c_\pi^2 g_A^2 W^2 \beta^2(W) Q^2 m_N^2}{4(W^2 - m_N^2)^2} \left(\frac{Q^2 G_M^2 + 4m_N^2 G_E^2}{Q^2 + 4m_N^2} \right) \right\}$$

$$g_1(W,Q^2) = \frac{\beta(W)}{(4\pi f_\pi)^2} \sum_{\pi^0,\pi^+} \left\{ \frac{Q^2}{2m_N^4} \left[|Q^2 G_1^{\pi N}|^2 - m_N^2 \mathrm{Re}(Q^2 G_1^{\pi N} G_2^{*,\pi N}) \right] + \frac{c_\pi^2 g_A^2 W^2 \beta^2(W)}{8(W^2 - m_N^2)^2} Q^2 m_N^2 G_M F_1^p \right\}$$

$$g_2(W,Q^2) = -\frac{\beta(W)}{(4\pi f_\pi)^2} \sum_{\pi^0,\pi^+} \left\{ \frac{Q^2}{2m_N^4} \left[|Q^2 G_1^{\pi N}|^2 + \frac{1}{4} Q^2 \mathrm{Re}(Q^2 G_1^{\pi N} G_2^{*,\pi N}) \right] + \frac{c_\pi^2 g_A^2 W^2 \beta^2(W)}{32(W^2 - m_N^2)^2} Q^4 G_M F_2^p \right\}$$

$$\beta(W) = \frac{|\vec{k_f}|}{W}, \qquad x_B = \frac{Q^2}{Q^2 + W^2 - m_N^2}$$

SLAC E136

P. E. Bosted *et al.*; PRD49(1994)3091

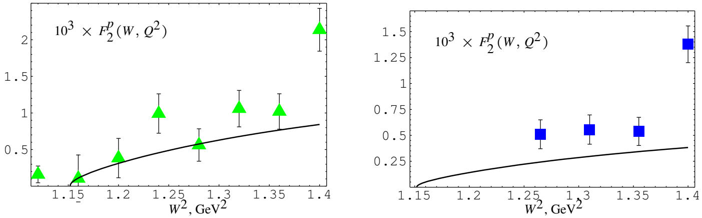


Figure: The structure function $F_2^p(W, Q^2)$ as a function of W^2 scaled by a factor 10^3 compared to the SLAC E136 data at the average value $Q^2 = 7.14$ GeV^2 (left panel) and $Q^2 = 9.43$ GeV^2 (right panel).

Differential Cross Section

For unpolarized protons, the virtual photon cross section is

$$d\sigma_{\gamma*} = \frac{\alpha_{\rm em}}{8\pi}\frac{k_f}{W}\frac{d\Omega_\pi}{W^2 - m_N^2}|\mathcal{M}_{\gamma*}|^2$$

with

$$|\mathcal{M}_{\gamma*}|^2 = M_T + \epsilon\, M_L + \sqrt{2\epsilon(1+\epsilon)}\, M_{LT}\cos(\phi_\pi) + \epsilon M_{TT}\cos(2\phi_\pi) + \lambda\sqrt{2\epsilon(1-\epsilon)}\, M'_{LT}\sin(\phi_\pi)$$

$$\begin{aligned}
f_\pi^2 M_T &= \frac{4\vec{k}_i^2 Q^2}{m_N^2}|G_1^{\pi N}|^2 + \frac{c_\pi^2 g_A^2 \vec{k}_f^2}{(W^2-m_N^2)^2} Q^2 m_N^2 G_M^2 + \cos\theta \frac{c_\pi g_A |k_i||k_f|}{W^2-m_N^2} 4Q^2 G_M \mathrm{Re}\, G_1^{\pi N} \\
f_\pi^2 M_L &= \vec{k}_i^2 |G_2^{\pi N}|^2 + \frac{4c_\pi^2 g_A^2 \vec{k}_f^2}{(W^2-m_N^2)^2} m_N^4 G_E^2 - \cos\theta \frac{c_\pi g_A |k_i||k_f|}{W^2-m_N^2} 4m_N^2 G_E \mathrm{Re}\, G_2^{\pi N} \\
f_\pi^2 M_{LT} &= -\sin\theta \frac{c_\pi g_A |k_i||k_f|}{W^2-m_N^2} Q m_N \left[G_M \mathrm{Re}\, G_2^{\pi N} + 4G_E \mathrm{Re}\, G_1^{\pi N}\right] \\
f_\pi^2 M_{TT} &= 0\,, \\
f_\pi^2 M'_{LT} &= -\sin\theta \frac{c_\pi g_A |k_i||k_f|}{W^2-m_N^2} Q m_N \left[G_M \mathrm{Im}\, G_2^{\pi N} - 4G_E \mathrm{Im}\, G_1^{\pi N}\right]
\end{aligned}$$

- ◇ $M_{TT} = 0$**: no D–wave; tests quality of the approximation**
- ◇ M'_{LT}**: single–spin asymmetry; arises because of FSI, calculable**

Miscellaneous Results

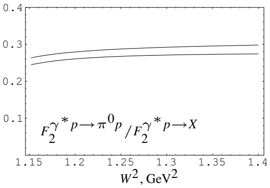


Figure: **The fraction of $\pi^0 p$ in $F_2^p(W,Q^2)$ for $Q^2 = 3$ GeV^2 (upper curve) and $Q^2 = 9$ GeV^2 (lower curve)**

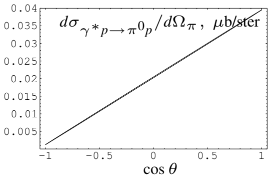


Figure: **Differential cross section $d\sigma_{\gamma^* p\to\pi^0 p}/d\Omega_\pi$ for $\phi_\pi = 135$ grad, $Q^2 = 4.2$ GeV^2 and $W = 1.11$ GeV**

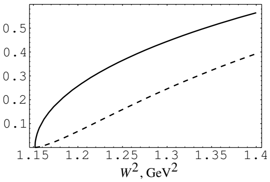


Figure: **S-wave (solid) vs. P-wave (dashed) for $F_2^p(W,Q^2)$ at $Q^2 = 7.14$ GeV^2**

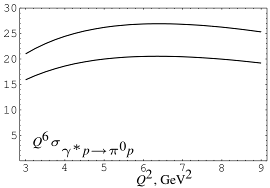


Figure: **Integrated cross section $Q^6\sigma_{\gamma^* p\to\pi^0 p}$ for $W = 1.11$ GeV (lower curve) and $W = 1.15$ GeV (upper curve)**

Preliminary Results from CLAS at Jefferson Lab

were shown at 4th Workshop on Exclusive Reactions at High Momentum Transfer, May, 2010 by Puneet Khetarpal for $\pi^0 p$ and Kijun Park for $\pi^+ n$ production

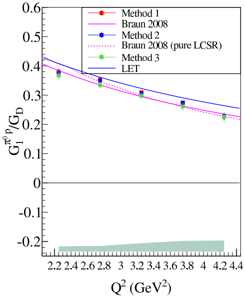


Figure: **Extracted $G_1^{\pi^0 p}/G_D$ ratio versus LCSR and LET predictions.**

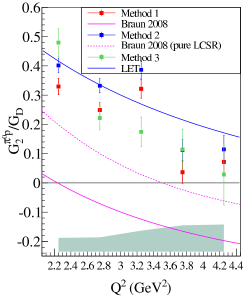


Figure: **Extracted $G_2^{\pi^0 p}/G_D$ ratio versus LCSR and LET predictions.**

Outlook

physics issues:

- **a novel object: generalized form factor; a complete direct measurement is possible**
- **interesting interplay between perturbative QCD and chiral symmetry breaking**
- **new connection between QCD and traditional hadron physics (PWA)**
- **an (almost) untouched terrain. . .**

theory:

- **detailed predictions possible near to threshold**
- **present LCSR accuracy probably 50%; can be improved to 20%(?)**
- **elastic πN rescattering can be taken into account**

experiment:

- **Preliminary results from CLAS at Jefferson Lab:**
  for $W = 1.09 - 1.23$ GeV, $Q^2 = 2.3 - 4.3\ \mathrm{GeV}^2$

Perfectly suited for the JLab 12 GeV upgrade physics program