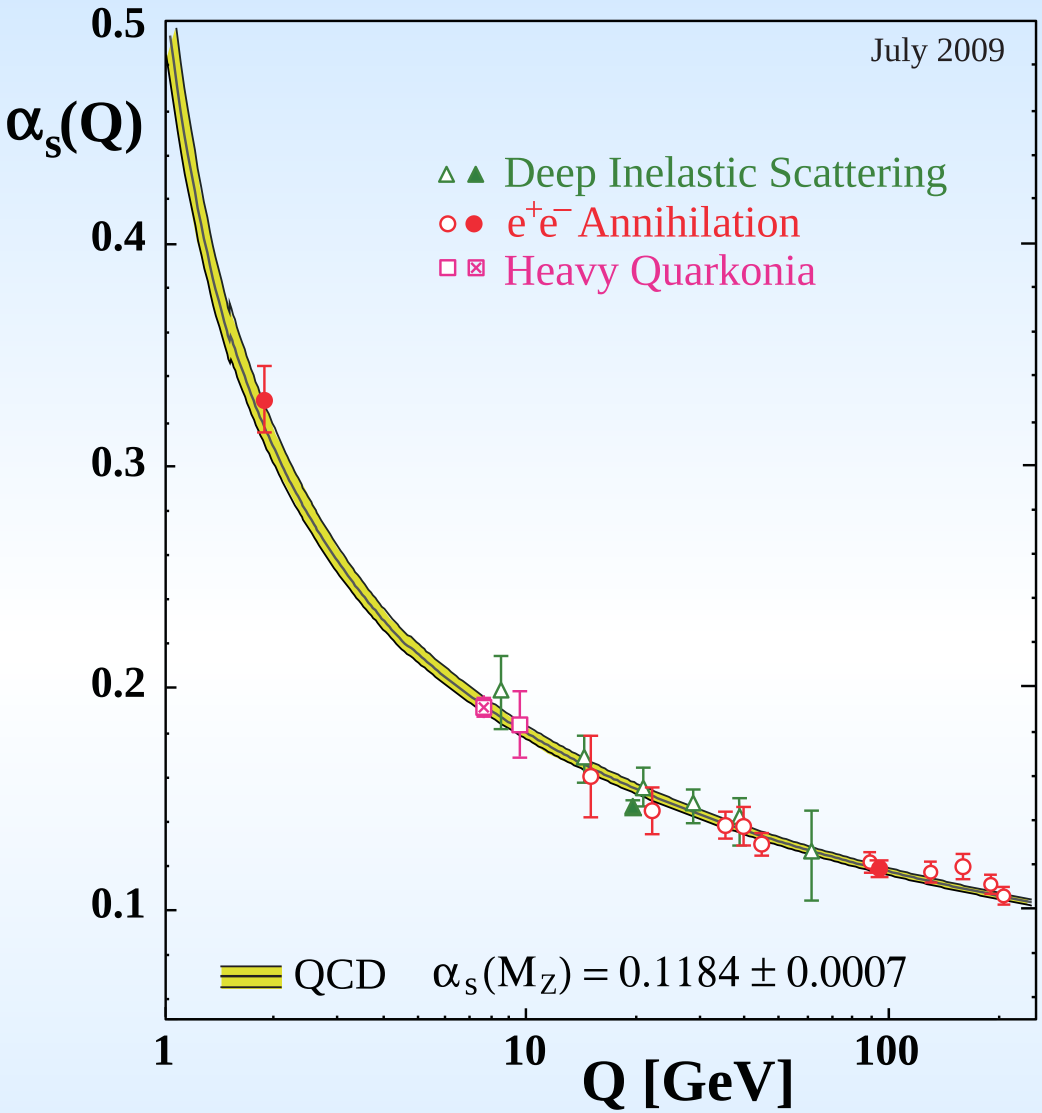
Determination of α_s from τ's

Matthias Jamin

α_s measurements



S. Bethke

For 0.6 % precision at M_Z need "only" ≈ 2 % at M_τ.

Consider the physical quantity R_τ: (Braaten, Narison, Pich 1992)

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \mathrm{hadrons}\,\nu_\tau(\gamma))}{\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau(\gamma))} = 3.6380(83)\,. \quad \text{(HFAG 2011)}$$

R_τ is related to the QCD correlators $\Pi^{(1,0)}(z)$: $(z \equiv s/M_\tau^2)$

$$R_\tau = 12\pi \int\limits_0^1 dz\,(1-z)^2 \Big[(1+2z)\,\mathrm{Im}\Pi^{(1)}(z) + \mathrm{Im}\Pi^{(0)}(z)\Big]\,,$$

with the appropriate combinations

$$\Pi^{(J)}(z) = |V_{ud}|^2 \Big[\Pi_{ud}^{V,J} + \Pi_{ud}^{A,J}\Big] + |V_{us}|^2 \Big[\Pi_{us}^{V,J} + \Pi_{us}^{A,J}\Big]\,.$$

Hadronic τ decay moments

Additional exp information can be inferred from the moments

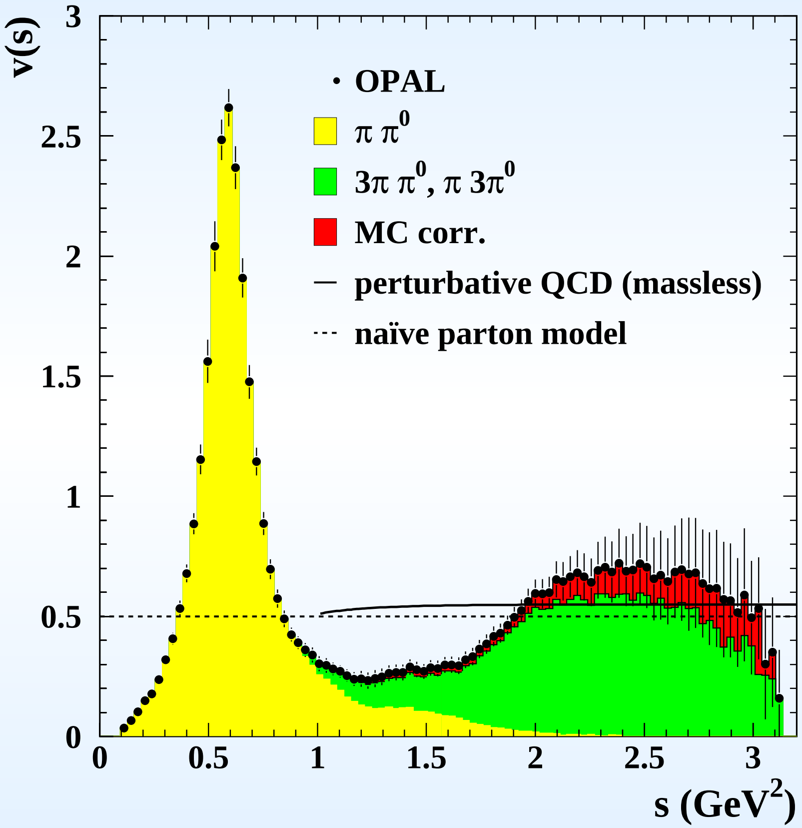
$$R_\tau^w \equiv \int_0^1 dz\, w(z)\, \frac{dR_\tau}{dz} = R_{\tau,V}^w + R_{\tau,A}^w + R_{\tau,S}^w\,.$$

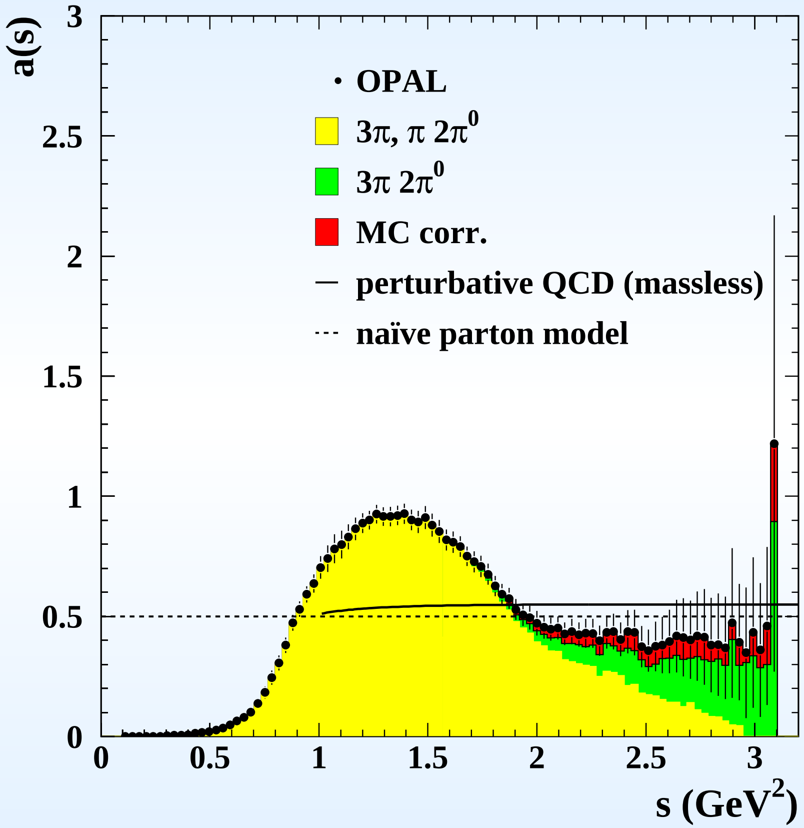
Theoretically, R_τ^w can be expressed as:

$$R_\tau^w = N_c\, S_{\rm EW} \Big\{ (|V_{ud}|^2 + |V_{us}|^2) \Big[1 + \delta^{w(0)} \Big] + \sum_{D\geq 2} \Big[|V_{ud}|^2 \delta_{ud}^{w(D)} + |V_{us}|^2 \delta_{us}^{w(D)} \Big] \Big\}\,.$$

$\delta_{ud}^{w(D)}$ and $\delta_{us}^{w(D)}$ are corrections in the Operator Product Expansion, the most important ones being $\sim m_s^2$ and $m_s \langle \bar{q} q \rangle$.

OPAL 1998 V&A spectra





FOPT vs CIPT problem

The purely perturbative contribution $\delta^{(0)}$ is plagued by differences for different RG-resummations. (FOPT vs CIPT.)

Using $\alpha_s(M_\tau) = 0.3186$, the numerical analysis results in:

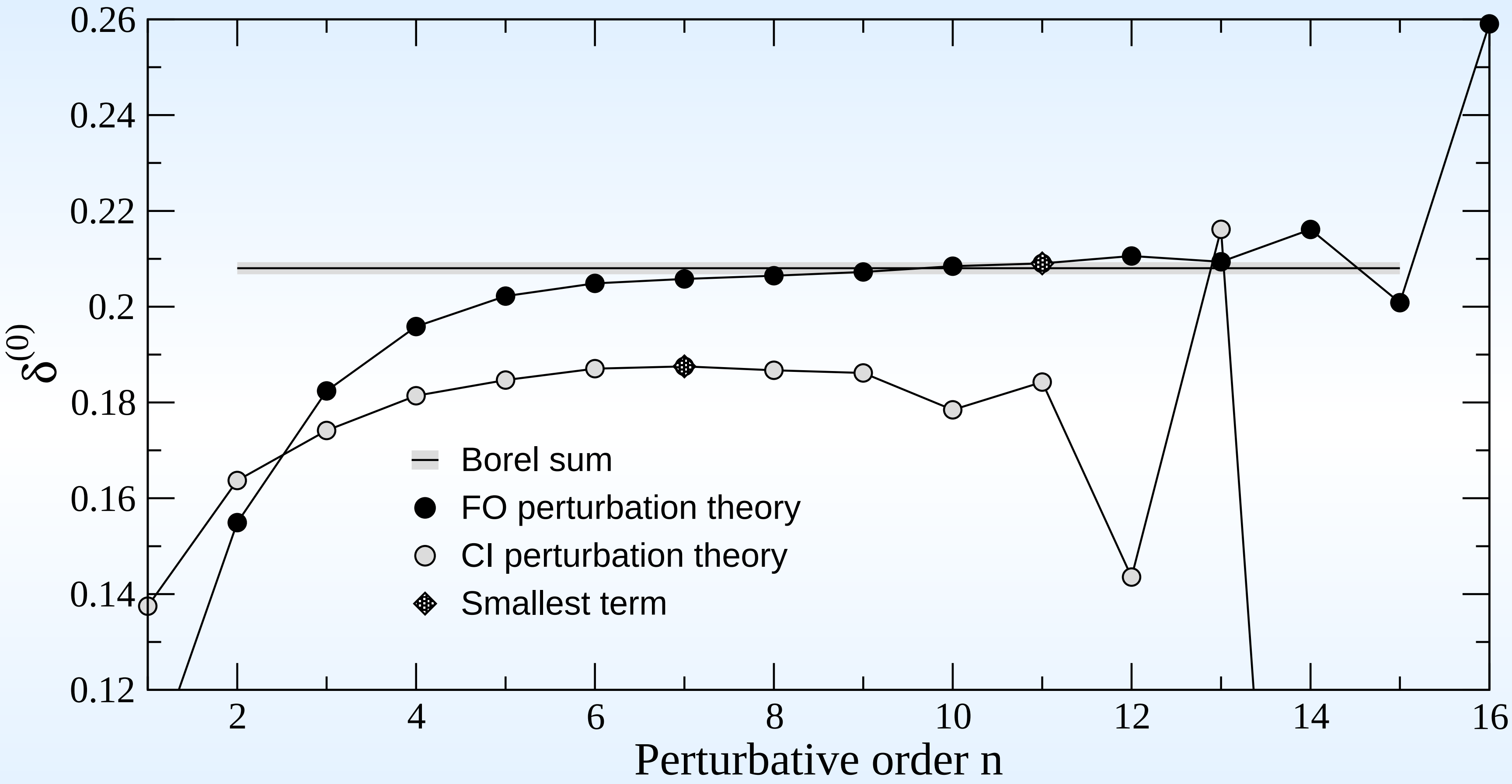
$$
\begin{array}{lccccc}
 & a^1 & a^2 & a^3 & a^4 & a^5 \\
\delta^{(0)}_{\rm FO} = & 0.101 + & 0.054 + & 0.027 + & 0.013 & (+0.006) = 0.196\,(0.202) \\
\delta^{(0)}_{\rm CI} = & 0.137 + & 0.026 + & 0.010 + & 0.007 & (+0.003) = 0.181\,(0.185)
\end{array}
$$

Contour improved PT appears to be better convergent.

The difference between both approaches is $0.015\,(0.017)$!

This problematic entails a ≈ 6 % difference for $\alpha_s(M_\tau)$.

Central Borel model



$c_{5,1} = 283\,, \quad \alpha_s(M_\tau) = 0.3186\,.$ (Beneke, MJ 2008)

Model for DV's

In the OPE, close to the Minkowskian axis ($s > 0$), so-called Duality Violations (DV's) can appear.

They can be studied on the basis of a toy-model:

(Shifman et al. 1995-2000)

(Catà, Golterman, Peris 2005/2008)

$$\Pi_V(s) = -\,\psi\left(\frac{M_V^2 + u(s)}{\Lambda^2}\right) + \text{const.}\,.$$

where

$$u(s) = \Lambda^2 \left(\frac{-s}{\Lambda^2}\right)^{\zeta} \quad \text{and} \quad \zeta \;=\; 1 - \frac{a}{\pi N_c}\,.$$

The model is based on large-N_c QCD and Regge-theory.

$$M_V = 770\,\mathrm{MeV}, \quad \Lambda = 1.2\,\mathrm{GeV}, \quad a = 0.4\,.$$

The OPE corresponds to the asymptotic expansion of the ψ-function for large s (large u).

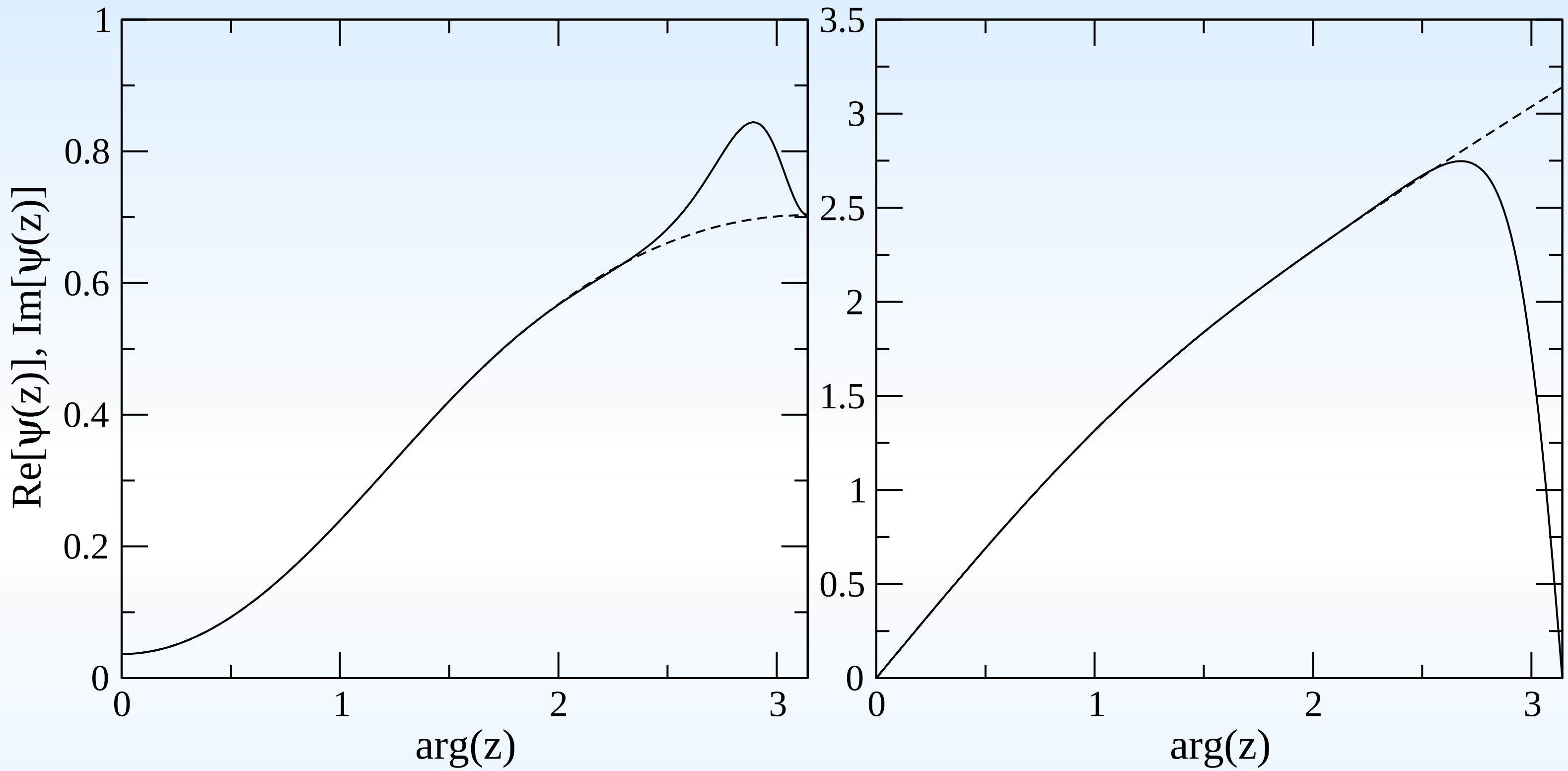
$$\psi(z) \sim \ln z - \frac{1}{2z} - \sum_{n=1}^{\infty} \frac{B_{2n}}{2nz^{2n}}\,, \qquad \mathrm{Re}z > 0\,.$$

In the Minkowskian region, an additional term arises:

$$-\,\pi\,[\,\cot\,(\pi z) \pm i\,]\,, \qquad \mathrm{Re}z < 0\,, \;\; \mathrm{Im}z \gtrless 0\,.$$

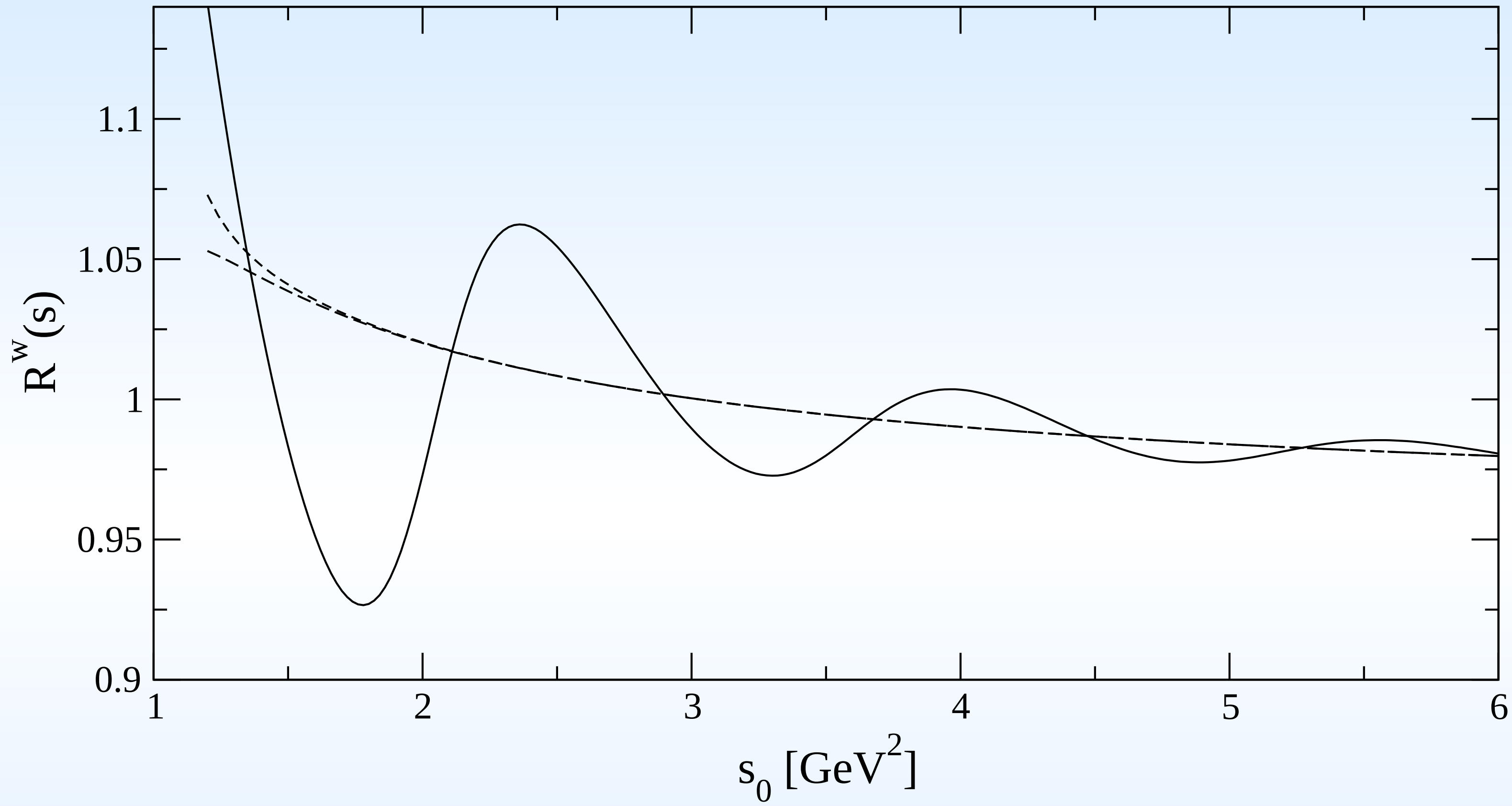
Formally, this term is exponentially suppressed, but it is enhanced by the poles of the ψ-function.

Asymptotic expansion



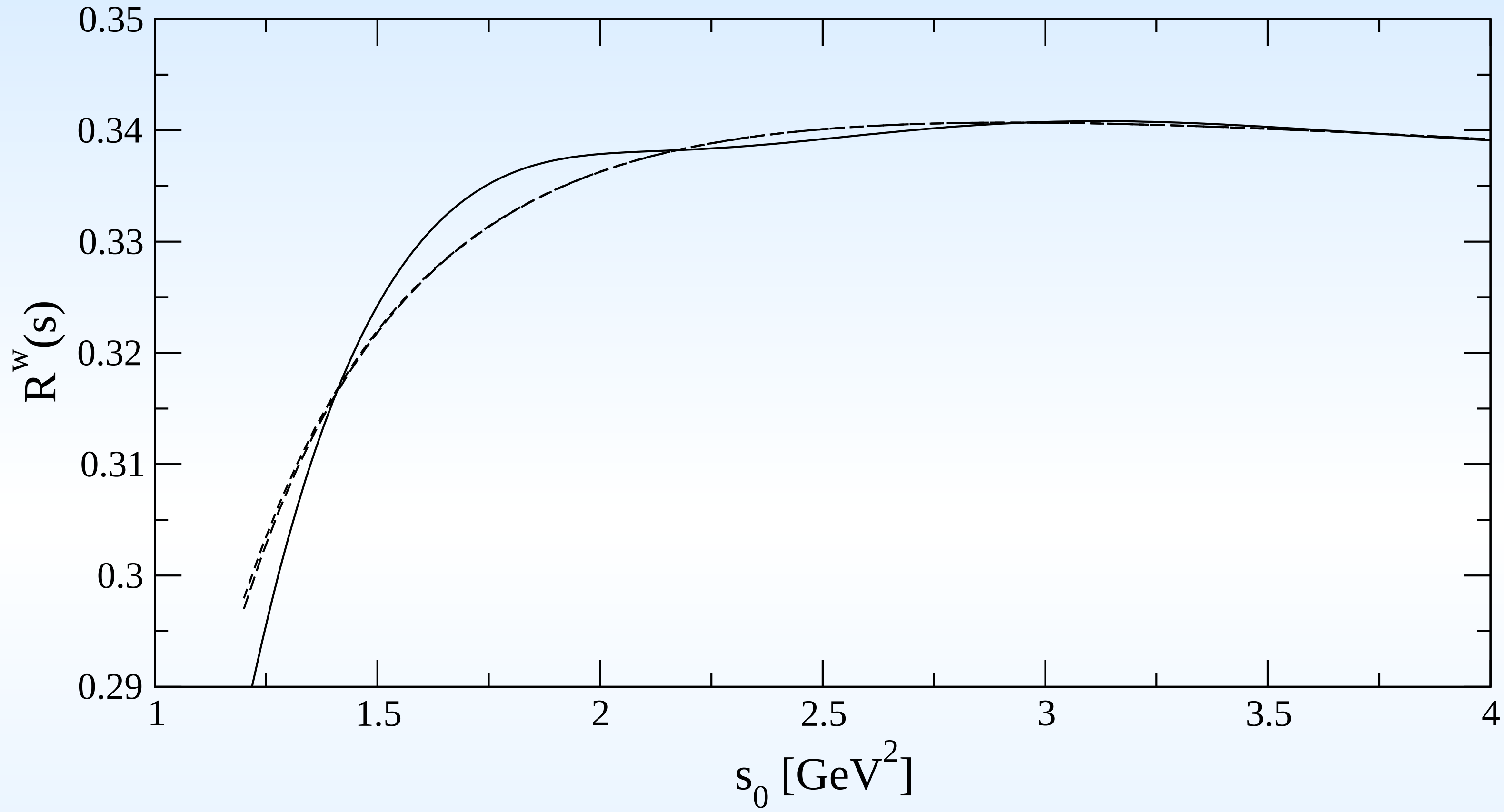
$z = 1.5 \cdot \exp(i\varphi)$ (MJ 2011)

ψ-function moments



ψ-function moment for $w(z) = 1$. (MJ 2011)

ψ-function moments



ψ-function moment for $w(z) = (1 - z)^2$.

Fits to OPAL data

In fits to experimental data, a model for DV's should be included.

The ψ-function model suggests an oscillating, decaying exponential, which can be chosen of the form:

$$\rho^{\rm DV}_{V/A}(s) = \kappa_{V/A}\, {\rm e}^{-\gamma_{V/A} s} \sin\left(\alpha_{V/A} + \beta_{V/A} s\right) .$$

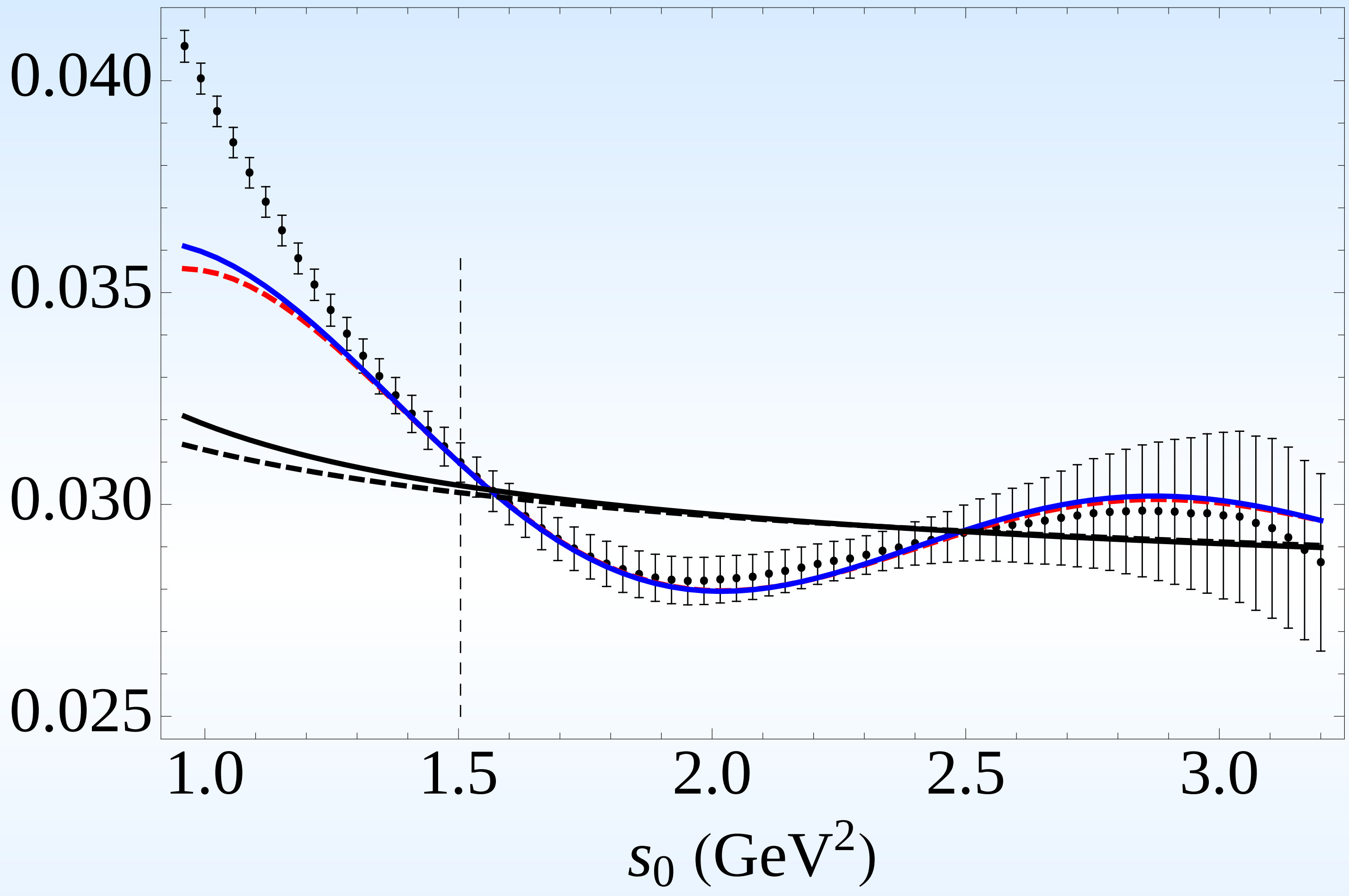
The fit quantities are the w-moments of the exp spectra.

$$R^w_{\tau,V/A}(s_0) \equiv \int\limits_0^{s_0} ds\, w(s)\, \rho_{V/A}(s) .$$

The cleanest moment turns out to be $w(s) = 1$.

Fitting combinations of several moments is complicated by very strong correlations.

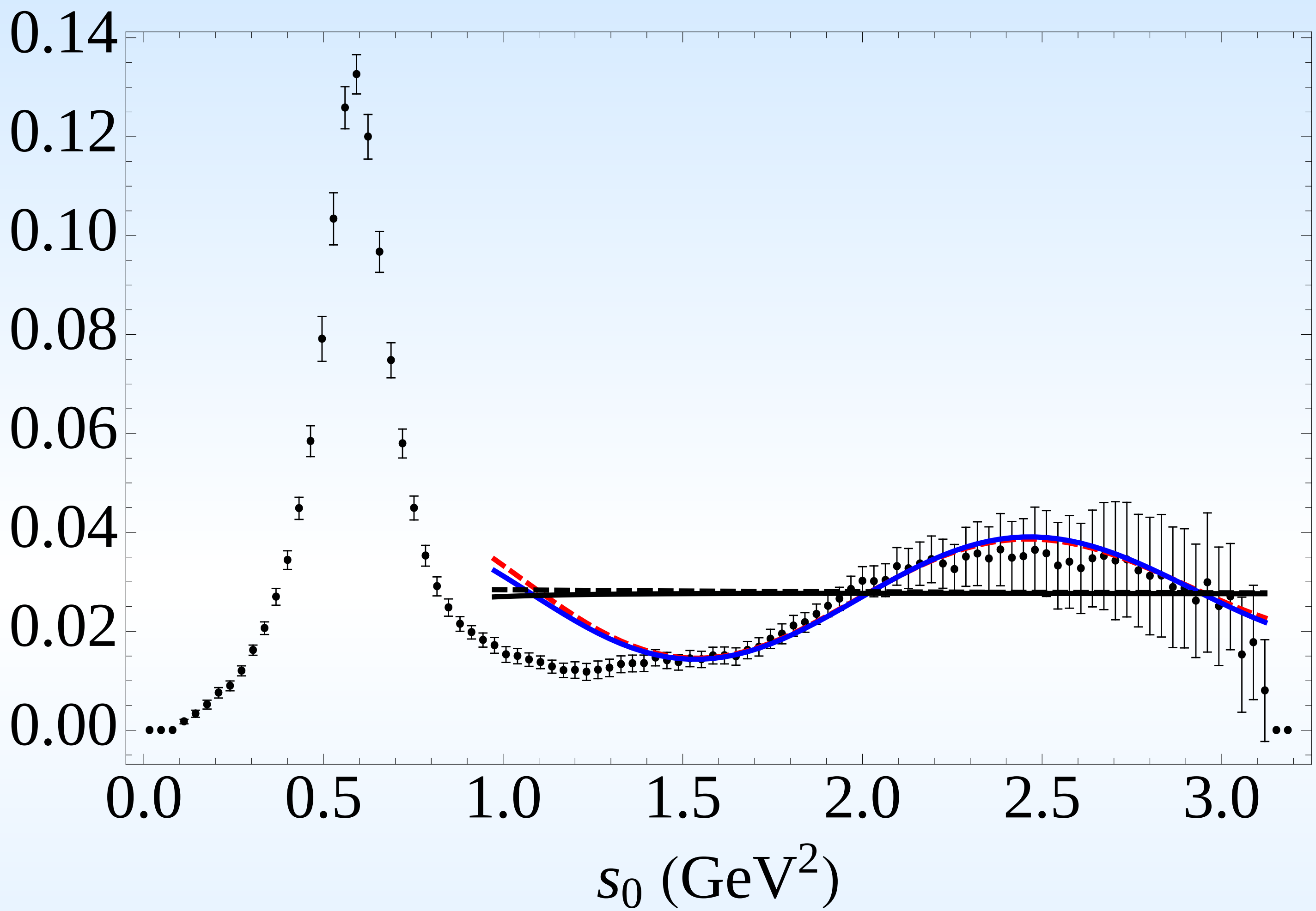
OPAL V-moment



$w(s) = 1$ (preliminary)

(Boito, Catà, Golterman, MJ, Maltman, Osborne, Peris 2011)

OPAL V-spectrum



$w(s) = 1$ (preliminary)

(Boito, Catà, Golterman, MJ, Maltman, Osborne, Peris 2011)

Conclusions

- Presently, the most reliable value of α_s from τ's including DV's comes from the trivial moment $w(s) = 1$.

$$\Rightarrow \quad \alpha_s(M_\tau) = 0.307 \pm 0.018 \pm 0.004 \quad \text{(FOPT)}$$

$$\Rightarrow \quad \alpha_s(M_\tau) = 0.322 \pm 0.025 \pm 0.004 \quad \text{(CIPT)}$$

- These values should be compared to the World Average (Bethke 2009): $\alpha_s(M_\tau) = 0.3186 \pm 0.0058$.

- Better data on exclusive and inclusive τ decay spectra would be helpful to resolve theoretical issues.

Conclusions

- Presently, the most reliable value of α_s from τ's including DV's comes from the trivial moment $w(s)=1$.

$$\Rightarrow \quad \alpha_s(M_\tau) = 0.307 \pm 0.018 \pm 0.004 \quad \text{(FOPT)}$$

$$\Rightarrow \quad \alpha_s(M_\tau) = 0.322 \pm 0.025 \pm 0.004 \quad \text{(CIPT)}$$

- These values should be compared to the World Average (Bethke 2009): $\alpha_s(M_\tau) = 0.3186 \pm 0.0058$.

- Better data on exclusive and inclusive τ decay spectra would be helpful to resolve theoretical issues.

Thank You for Your attention !