Determination of $\alpha_s$ from $\tau$'s

Matthias Jamin
$\alpha_s$ measurements

For 0.6% precision at $M_Z$ need “only” $\approx 2\%$ at $M_\tau$.

Determination of $\alpha_s$ from $\tau$’s
Matthias Jamin, ICREA & IFAE, UA Barcelona

$\Phi \to \Psi$, Novosibirsk 2011
Consider the physical quantity $R_\tau$: (Braaten, Narison, Pich 1992)

$$R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \text{hadrons} \, \nu_\tau(\gamma))}{\Gamma(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau(\gamma))} = 3.6380(83). \ (\text{HFAG 2011})$$

$R_\tau$ is related to the QCD correlators $\Pi^{(1,0)}(z)$: ($z \equiv s/M^2_\tau$)

$$R_\tau = 12\pi \int_0^1 dz (1-z)^2 \left[ (1+2z) \text{Im}\Pi^{(1)}(z) + \text{Im}\Pi^{(0)}(z) \right],$$

with the appropriate combinations

$$\Pi^{(J)}(z) = |V_{ud}|^2 \left[ \Pi^{V,J}_{ud} + \Pi^{A,J}_{ud} \right] + |V_{us}|^2 \left[ \Pi^{V,J}_{us} + \Pi^{A,J}_{us} \right].$$
Additional exp information can be inferred from the moments

\[ R_w^\tau \equiv \frac{1}{0} \int dz \ w(z) \ \frac{dR_\tau}{dz} = R_{\tau,V}^w + R_{\tau,A}^w + R_{\tau,S}^w. \]

Theoretically, \( R_w^\tau \) can be expressed as:

\[
R_w^\tau = N_c \ S_{\text{EW}} \left\{ (|V_{ud}|^2 + |V_{us}|^2) \left[ 1 + \delta^{w(0)} \right] \right. \\
+ \sum_{D \geq 2} \left[ |V_{ud}|^2 \delta^{w(D)}_{ud} + |V_{us}|^2 \delta^{w(D)}_{us} \right] \right\}.
\]

\( \delta^{w(D)}_{ud} \) and \( \delta^{w(D)}_{us} \) are corrections in the Operator Product Expansion, the most important ones being \( \sim m_s^2 \) and \( m_s \langle \bar{q}q \rangle \).
Determination of $\alpha_s$ from $\tau$'s
Matthias Jamin, ICREA & IFAE, UA Barcelona

$\Phi \rightarrow \Psi$, Novosibirsk 2011
The purely perturbative contribution $\delta^{(0)}$ is plagued by differences for different RG-resummations. (FOPT vs CIPT.)

Using $\alpha_s(M_\tau) = 0.3186$, the numerical analysis results in:

$$
\begin{align*}
\delta^{(0)}_{\text{FO}} &= a^1 + a^2 + a^3 + a^4 + a^5 = 0.101 + 0.054 + 0.027 + 0.013 (+0.006) = 0.196 (0.202) \\
\delta^{(0)}_{\text{CI}} &= 0.137 + 0.026 + 0.010 + 0.007 (+0.003) = 0.181 (0.185)
\end{align*}
$$

Contour improved PT appears to be better convergent.

The difference between both approaches is $0.015 (0.017)$!

This problematic entails a $\approx 6\%$ difference for $\alpha_s(M_\tau)$. 

Matthias Jamin, ICREA & IFAE, UA Barcelona

Φ → Ψ, Novosibirsk 2011
$c_{5,1} = 283$, $\alpha_s(M_\tau) = 0.3186$. (Beneke, MJ 2008)
In the OPE, close to the Minkowskian axis ($s > 0$), so-called Duality Violations (DV’s) can appear.

They can be studied on the basis of a toy-model:

\[
\Pi_V(s) = -\psi\left(\frac{M_V^2 + u(s)}{\Lambda^2}\right) + \text{const. .}
\]

where

\[
u(s) = \Lambda^2 \left(\frac{-s}{\Lambda^2}\right) \zeta \quad \text{and} \quad \zeta = 1 - \frac{a}{\pi N_c}.
\]

The model is based on large-$N_c$ QCD and Regge-theory.

\[
M_V = 770 \text{ MeV}, \quad \Lambda = 1.2 \text{ GeV}, \quad a = 0.4.
\]
The OPE corresponds to the asymptotic expansion of the $\psi$-function for large $s$ (large $u$).

$$\psi(z) \sim \ln z - \frac{1}{2z} - \sum_{n=1}^{\infty} \frac{B_{2n}}{2n z^{2n}}, \quad \text{Re}z > 0.$$ 

In the Minkowskian region, an additional term arises:

$$- \pi \left[ \cot (\pi z) \pm i \right], \quad \text{Re}z < 0, \; \text{Im}z \geq 0.$$ 

Formally, this term is exponentially suppressed, but it is enhanced by the poles of the $\psi$-function.
\[ z = 1.5 \cdot \exp (i \varphi) \]  

(MJ 2011)
\( \psi \)-function moments

\[ \psi \text{-function moment for } w(z) = 1. \]  
(MJ 2011)
\( \psi \)-function moment for \( w(z) = (1 - z)^2 \).
In fits to experimental data, a model for DV’s should be included.

The $\psi$-function model suggests an oscillating, decaying exponential, which can be chosen of the form:

$$\rho_{V/A}^{DV}(s) = \kappa_{V/A} e^{-\gamma_{V/A}s} \sin \left( \frac{\alpha_{V/A}}{A} + \frac{\beta_{V/A}}{A}s \right).$$

The fit quantities are the $w$-moments of the exp spectra.

$$R_{\tau,V/A}^{w}(s_0) \equiv \int_{0}^{s_0} ds \ w(s) \ \rho_{V/A}(s).$$

The cleanest moment turns out to be $w(s) = 1$.

Fitting combinations of several moments is complicated by very strong correlations.
\( w(s) = 1 \quad \text{(preliminary)} \)

(Boito, Catà, Golterman, MJ, Maltman, Osborne, Peris 2011)
$w(s) = 1$  (preliminary)

(Boito, Catà, Golterman, MJ, Maltman, Osborne, Peris 2011)
Presently, the most reliable value of $\alpha_s$ from $\tau$’s including DV’s comes from the trivial moment $w(s) = 1$.

\[ \Rightarrow \quad \alpha_s(M_\tau) = 0.307 \pm 0.018 \pm 0.004 \] (FOPT)

\[ \Rightarrow \quad \alpha_s(M_\tau) = 0.322 \pm 0.025 \pm 0.004 \] (CIPT)

These values should be compared to the World Average (Bethke 2009): $\alpha_s(M_\tau) = 0.3186 \pm 0.0058$.

Better data on exclusive and inclusive $\tau$ decay spectra would be very helpful to resolve theoretical issues.
Presently, the most reliable value of $\alpha_s$ from $\tau$'s including DV's comes from the trivial moment $w(s) = 1$.

$\Rightarrow \quad \alpha_s(M_\tau) = 0.307 \pm 0.018 \pm 0.004$ (FOPT)

$\Rightarrow \quad \alpha_s(M_\tau) = 0.322 \pm 0.025 \pm 0.004$ (CIPT)

These values should be compared to the World Average (Bethke 2009): $\alpha_s(M_\tau) = 0.3186 \pm 0.0058$.

Better data on exclusive and inclusive $\tau$ decay spectra would be very helpful to resolve theoretical issues.

Thank You for Your attention!