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Light scalars, analytical $\pi\pi$ scattering amplitude, the Roy equations, and chiral expansion

N.N. Achasov and A.V. Kiselev, Phys. Rev. **D83**, 054008 (2011)

We will concern the data on following processes with the birth of $\sigma(600)$ and $f_0(980)$:

1. $\pi\pi$**-scattering**

2. $\phi \to \pi^0\pi^0\gamma$

Chiral one-loop mechanism of the transition
$\phi \to K^+K^- \to [\sigma(600) + f_0(980)]\gamma$ **(Kaon Loop model),**
N.N. Achasov and V.N. Ivanchenko (1987)

Experiment:
- **discovery at BINP (1998)**
- **high-statistical data from KLOE (2002)**

3. $\pi\pi \to K\bar{K}$

2005. The KLOE data on the $\phi \to \pi^0\pi^0\gamma$ decay are described simultaneously with the data on the $\pi\pi$ scattering and the $\pi\pi \to K\bar{K}$ reaction. The description is carried out taking into account the chiral shielding of $\sigma(600)$ and the $\sigma(600) - f_0(980)$ mixing. The data don't contradict the existence of the $\sigma(600)$ meson and yield evidence in favor of the four-quark nature of the $\sigma(600)$ and $f_0(980)$ mesons.

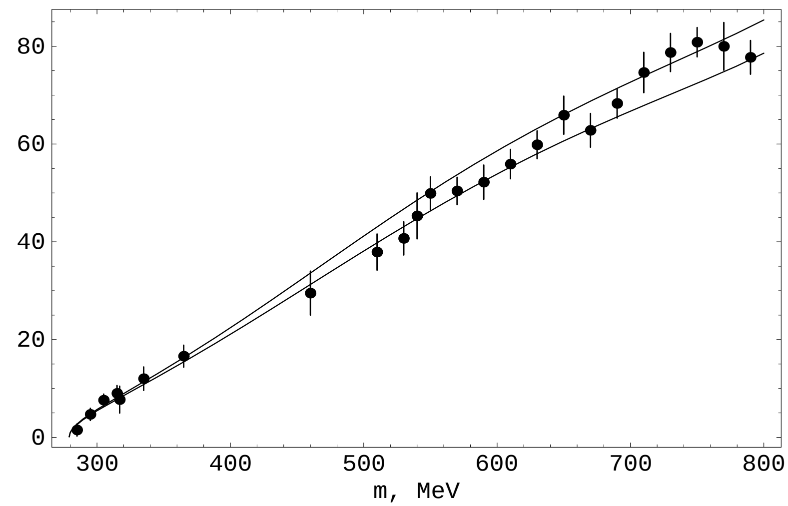
N. N. Achasov, A. V. Kiselev, Phys. Rev. D 73, 054029 (2006) [Erratum-ibid. D 74, 059902 (2006)];
Yad. Fiz. 70, 2005 (2007) [Phys. At. Nucl. 70, 1956 (2007)].

2006. **I. Caprini, G. Colangelo and H. Leutwyler, Phys. Rev. Lett. 96, 132001 (2006):**

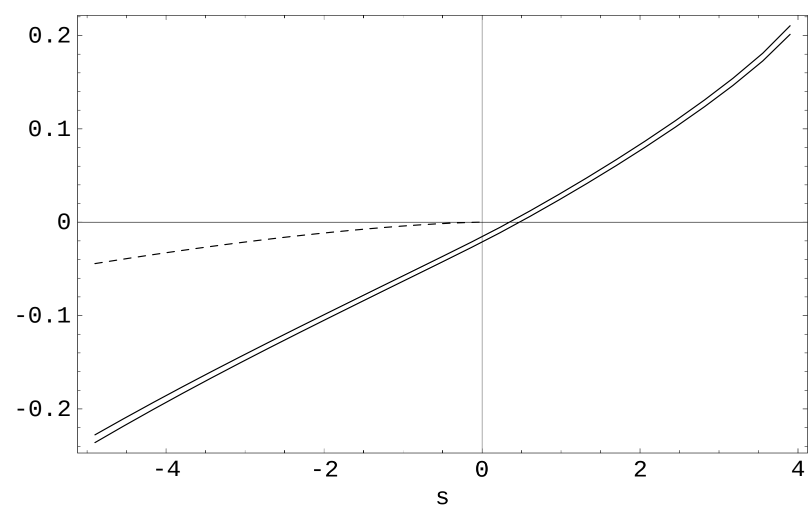
The $\pi\pi$ scattering amplitude is calculated in the s complex plane, the calculation is based on chiral expansion, dispersion relations and Roy equations.

The σ pole is at

$$M_\sigma = 441^{+16}_{-8} - i272^{+9}_{-12.5}\ \mathbf{MeV}$$



The corridor in the phase δ_0^0 of the $\pi\pi$ scattering



The real and the imaginary parts of the amplitude T_0^0 of the $\pi\pi$ scattering (s in units of m_π^2)

The S-wave amplitude T_0^0 of the $\pi\pi$ scattering with I=0 is

$$T_0^0 = \frac{\eta_0^0 e^{2i\delta_0^0} - 1}{2i\rho_{\pi\pi}(m)} = \frac{e^{2i\delta_B^{\pi\pi}} - 1}{2i\rho_{\pi\pi}(m)} + e^{2i\delta_B^{\pi\pi}} \sum_{R,R'} \frac{g_{R\pi\pi} G_{RR'}^{-1} g_{R'\pi\pi}}{16\pi}$$

$$G_{RR'}(m) = \begin{pmatrix} D_{f_0}(m) & -\Pi_{f_0\sigma}(m) \\ -\Pi_{f_0\sigma}(m) & D_\sigma(m) \end{pmatrix}$$

The desired analytical properties of the $\pi\pi$ scattering amplitude are: two cuts in the s-complex plane, Adler zero, absence of poles on the physical sheet of the Riemannian surface, resonance poles on the second sheet of the Riemannian surface. This applies curtain restrictions on the $\delta_B^{\pi\pi}$.

N.N. Achasov and A.V. Kiselev, Phys. Rev. D70, 111901 (R)(2004):

The inverse propagator of scalar R

$$D_R(m^2) = m_R^2 - m^2 + Re\left(\Pi_R(m_R^2)\right) - \Pi_R(m^2)$$

$$\Pi_R^{ab}(m^2) = \frac{g_{Rab}^2}{16\pi}\Pi^{ab}(m^2) = \frac{1}{\pi}[m^2 - (m_a + m_b)^2]\times$$

$$\int_{(m_a+m_b)^2}^{\infty} \frac{\bar{m}\Gamma(R \to ab, \bar{m})\, d\bar{m}^2}{[\bar{m}^2 - (m_a + m_b)^2](\bar{m}^2 - m^2 - i\varepsilon)}$$

$$Im\,(D_R(z)) = -y\left(1 + \sum_{ab} \frac{1}{\pi} \int_{(m_a+m_b)^2}^{\infty} \frac{\bar{m}\Gamma(R \to ab, \bar{m})}{|\bar{m}^2 - z|^2}\, d\bar{m}^2\right)$$

$$e^{2i\delta_B^{\pi\pi}} = \frac{P^*_{\pi 1}(s) P^*_{\pi 2}(s)}{P_{\pi 1}(s) P_{\pi 2}(s)}$$

$$P_{\pi 1}(s) = a_1 - a_2 \frac{s}{4m_\pi^2} - \Pi^{\pi\pi}(s) + a_3\, \Pi^{\pi\pi}(4m_\pi^2 - s) - a_4 Q_1(s)$$

$$Q_1(s) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{s - 4m_\pi^2}{s' - 4m_\pi^2} \frac{\rho_{\pi\pi}(s')}{s' - s - i\varepsilon} K_1(s')$$

$$Im(Q_1(s)) = K_1(s)\rho_{\pi\pi}(s)$$

Hence we require that $K_1(s)$ should be positive and have no singularities on the physical sheet.

$$K_1(s) = \frac{L_1(s)}{\prod_{i=1}^{6} D_i(4m_\pi^2 - s)} \tag{1}$$

$$L_1(s) = (s - 4m_\pi^2)^6 + \sum_{i=1}^{6} \alpha_i (s - 4m_\pi^2)^{6-i} + \sqrt{s} \sum_{i=1}^{6} c_i (s - 4m_\pi^2)^{6-i} \tag{2}$$

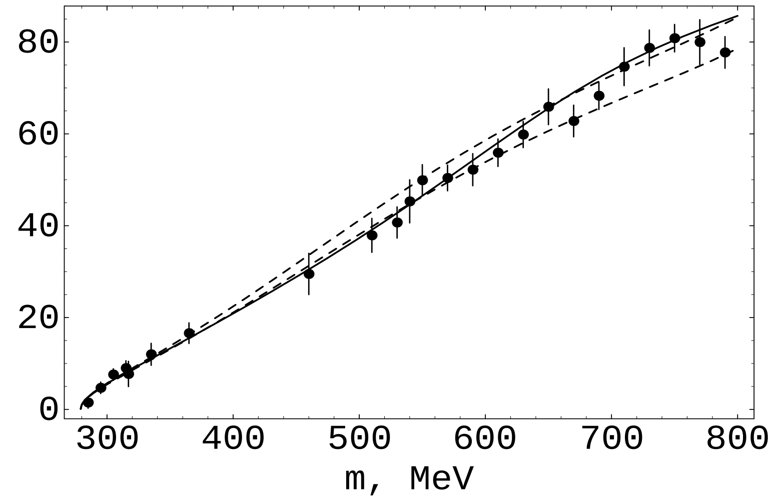
$$D_i(s) = m_i^2 - s - g_i \Pi^{\pi\pi}(s)$$

$$P_{\pi 2}(s) = \frac{\Lambda^2 + s - 4m_\pi^2}{4m_\pi^2} + k_2 Q_2(s)$$

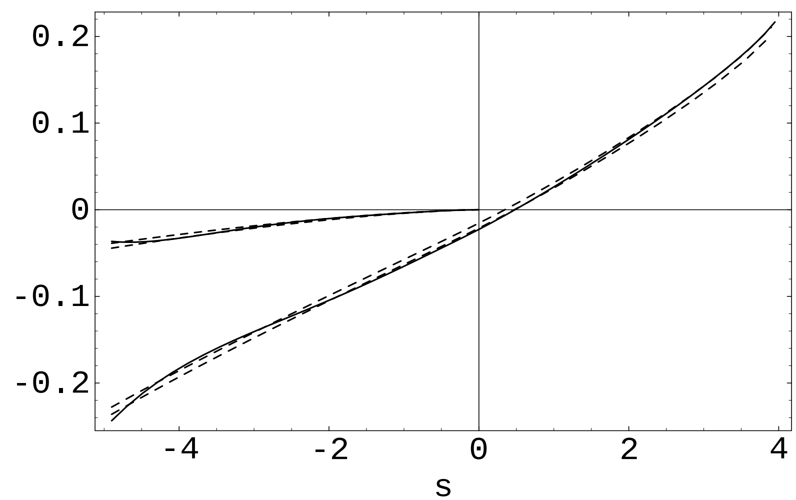
$$Q_2(s) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{s - 4m_\pi^2}{s' - 4m_\pi^2} \frac{\rho_{\pi\pi}(s')}{s' - s - i\varepsilon} K_2(s')$$

$$K_2(s) = \frac{L_2(s)}{D_{1A}(4m_\pi^2 - s) D_{2A}(4m_\pi^2 - s) D_{3A}(4m_\pi^2 - s)}$$

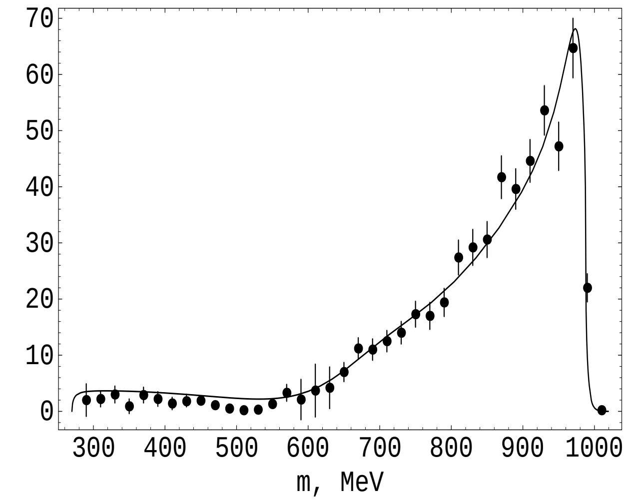
$$L_2(s) = 4m_\pi^2 (s^2 + \beta(4m_\pi^2)s + \gamma_1(2m_\pi)^3 s^{1/2} + \gamma_2(2m_\pi)s^{3/2})$$



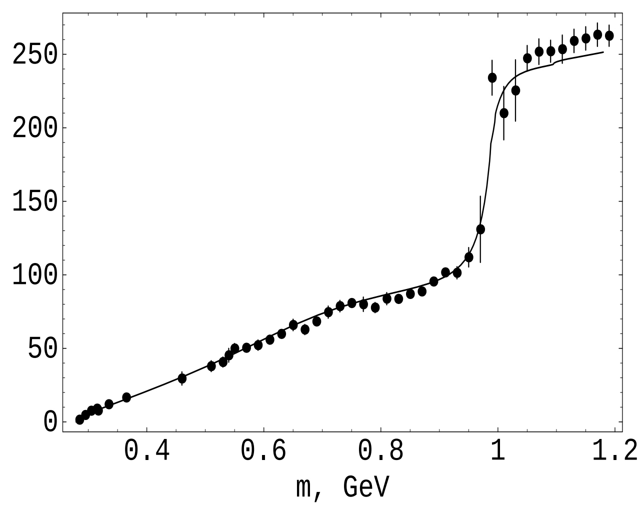
The phase δ_0^0 of the $\pi\pi$ scattering ($g^2_{f_0K^+K^-}/4\pi$=2 GeV^2)



The real and the imaginary parts of the amplitude T_0^0 of the $\pi\pi$ scattering (s in units of m_π^2)



The $\pi^0\pi^0$ spectrum in $\phi \to \pi^0\pi^0\gamma$ decay



The phase δ_0^0 of the $\pi\pi$ scattering

Poles of the resonances

$\sigma(600)$ poles (MeV) on different sheets of the complex s plane depending on lists of polarization operators $\Pi^{ab}(s)$

$\Pi^{K\bar{K}}$	$\Pi^{\eta\eta}$	$\Pi^{\eta\eta'}$	$\Pi^{\eta'\eta'}$	**Fit 1**	**Fit 2**
I	**I**	**I**	**I**	$581.0 - 212.7\,i$	$613.8 - 221.4\,i$
II	**I**	**I**	**I**	$617.5 - 353.0\,i$	$609.8 - 291.6\,i$
II	**II**	**I**	**I**	$554.3 - 375.3\,i$	$559.4 - 346.6\,i$
II	**II**	**II**	**I**	$579.0 - 475.2\,i$	$569.7 - 410.7\,i$
II	**II**	**II**	**II**	$625.7 - 474.9\,i$	$581.6 - 411.0\,i$

Contradicts $M_\sigma = 441^{+16}_{-8} - i272^{+9}_{-12.5}$ **MeV.**

$f_0(980)$ poles (MeV) on different sheets of the complex s plane depending on lists of polarization operators $\Pi^{ab}(s)$

$\Pi^{K\bar{K}}$	$\Pi^{\eta\eta}$	$\Pi^{\eta\eta'}$	$\Pi^{\eta'\eta'}$	**Fit 1**	**Fit 2**
I	**I**	**I**	**I**	$986.2 - 25.5\,i$	$990.5 - 19.4\,i$
II	**I**	**I**	**I**	$916.9 - 299.4\,i$	$1183.2 - 518.6\,i$
II	**II**	**I**	**I**	$966.8 - 450.5\,i$	$1366.0 - 756.5\,i$
II	**II**	**II**	**I**	$962.6 - 465.2\,i$	$1390.7 - 813.0\,i$
II	**II**	**II**	**II**	$962.5 - 608.0\,i$	$1495.6 - 1057.7\,i$

Summary

1. **The $\pi\pi$ scattering amplitude with correct analytical properties has been built.**
2. **This amplitude describes experimental data and the results based on Roy equations on the real s axis.**
3. **A contradiction in position of σ pole may be explained by important role of high channels in the $\pi\pi$ scattering amplitude and its analytical continuation even for $|s|$ much less than m_{f_0}.**
4. **The f_0 pole is situated far from the position, predicted by Breit-Wigner approximation.**
5. **New experiments are important for the study of light scalar mesons.**

Thank you for your attention!