

Modelling hadronic currents for $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**

Pablo Roig

Institut de Física d'Altes Energies (IFAE) Barcelona

In collaboration with Daniel Gómez Dumm, Antonio Pich, Olga Shekhovtsova, Tomasz Przedzinski and Zbigniew Was

PHI PSI 11: International Workshop on e^+e^- collisions from PHI PSI
Budker Institute of Nuclear Physics (BINP), Novosibirsk, Russia 19-22 September 2011

CONTENTS

- Introduction: Semileptonic τ decays

- Theoretical setting

- $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$

See Z. Was' talk for the description of the implementation of the MC program and many details interesting for the user

- Other two meson τ decay channels: $(K\pi)^-, K^- K^0$

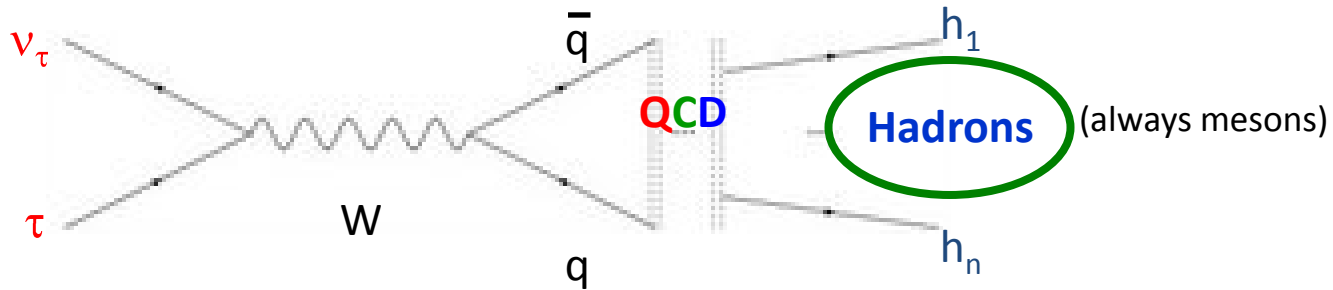
- Three meson τ decay channels: $(\pi\pi\pi)^-, (KK)\pi^-, K^- K^0 \pi^0$

- Conclusions and future work

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**

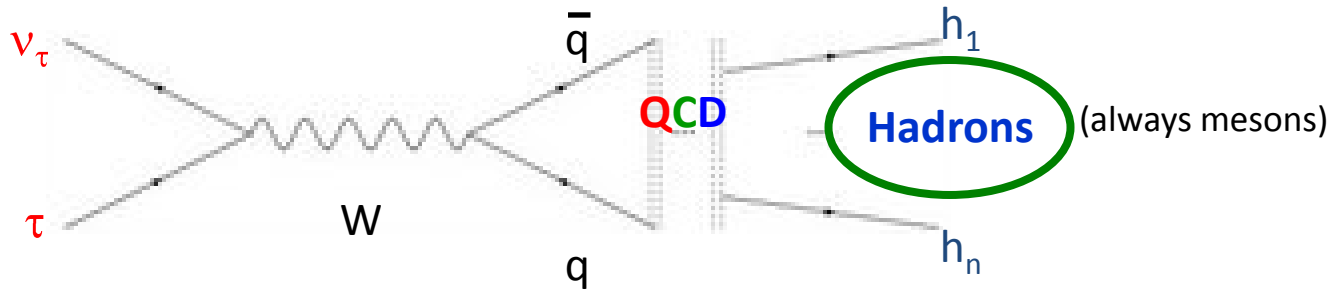
Introduction: Semileptonic τ decays



Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**

Introduction: Semileptonic τ decays



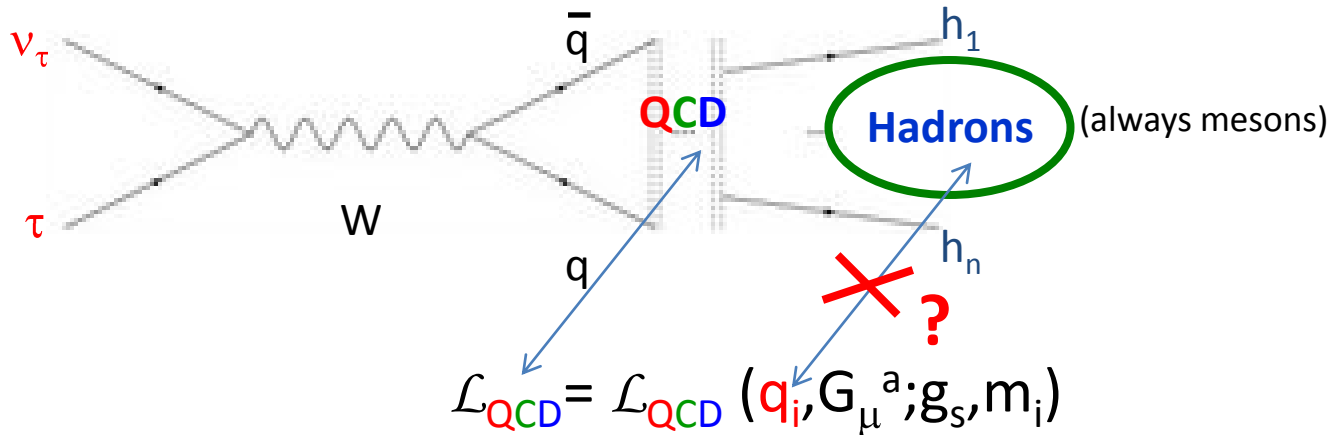
$$M = \frac{G_F}{\sqrt{2}} V_{CKM} \bar{u}(\nu_\tau) \gamma^\mu (1 - \gamma_5) u(\tau) T_\mu$$

$$T_\mu = \langle \text{Hadrons} | (\mathbf{V}-\mathbf{A})_\mu e^{iS_{\text{QCD}}} | 0 \rangle = \sum_i (\text{Lorentz Structure})^i F_i(Q^2, s_j)$$

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**

Introduction: Semileptonic τ decays



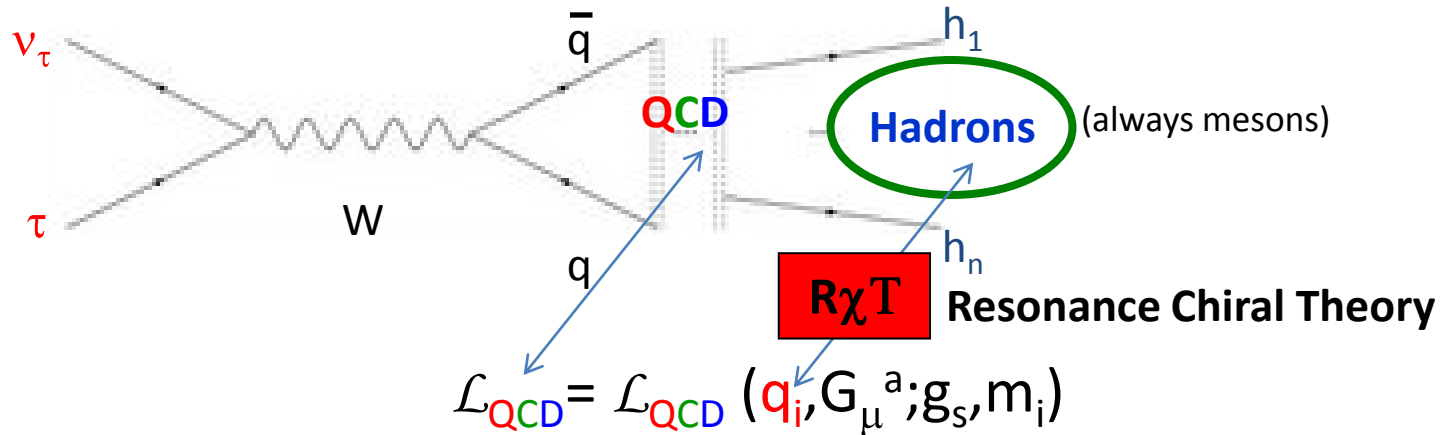
$$M = \frac{G_F}{\sqrt{2}} V_{CKM} \bar{u}(v_\tau) \gamma^\mu (1 - \gamma_5) u(\tau) T_\mu$$

$$T_\mu = \langle \text{Hadrons} | (\mathbf{V}-\mathbf{A})_\mu e^{iS_{\text{QCD}}} | 0 \rangle = \sum_i (\text{Lorentz Structure})^i F_i(Q^2, s_j)$$

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**

Introduction: Semileptonic τ decays



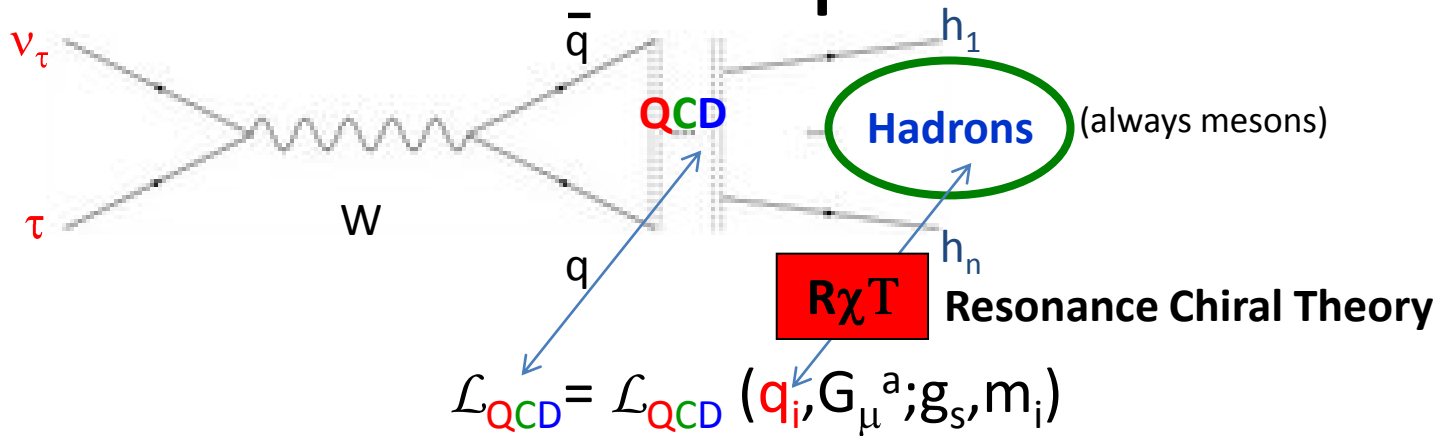
$$M = \frac{G_F}{\sqrt{2}} V_{CKM} \bar{u}(v_\tau) \gamma^\mu (1 - \gamma_5) u(\tau) T_\mu$$

$$T_\mu = \langle \text{Hadrons} | (\mathbf{V}-\mathbf{A})_\mu e^{iS_{\text{QCD}}} | 0 \rangle = \sum_i (\text{Lorentz Structure})^i F_i(Q^2, s_j)$$

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**

Introduction: Semileptonic τ decays



$$M = \frac{G_F}{\sqrt{2}} V_{CKM} \bar{u}(v_\tau) \gamma^\mu (1 - \gamma_5) u(\tau) T_\mu$$

$$T_\mu = \langle \text{Hadrons} | (\mathbf{V}-\mathbf{A})_\mu e^{iS_{\text{QCD}}} | 0 \rangle = \sum_i (\text{Lorentz Structure})^i F_i(Q^2, s_j)$$

Two mesons $h_1(p_1), h_2(p_2)$:

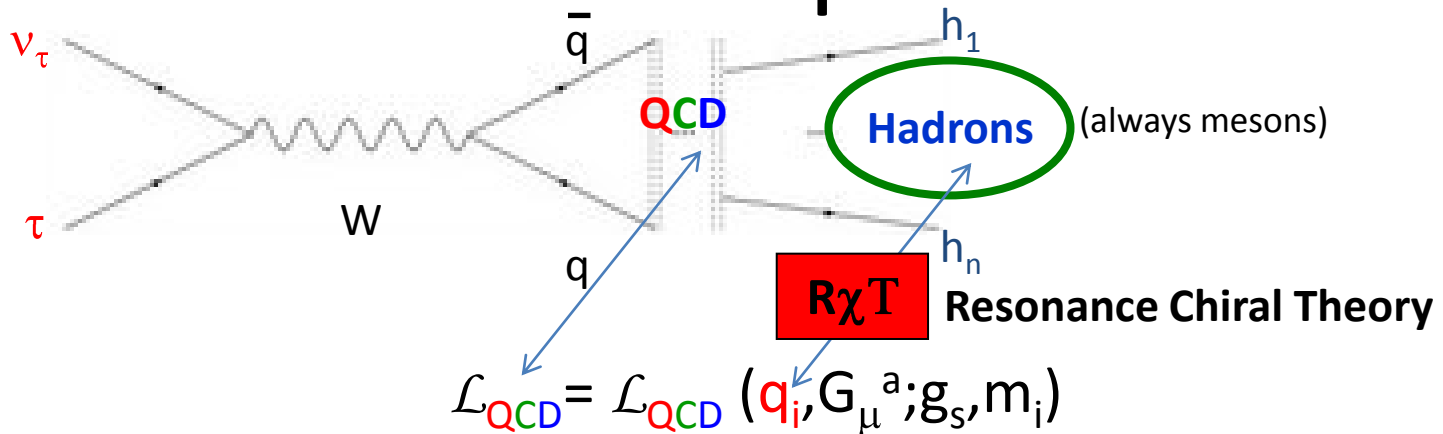
$$J^\mu = N [(p_1 - p_2)^\mu F^V(s) + (p_1 + p_2)^\mu F^S(s)]$$

$$(T_\mu \sim J_\mu)$$

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**

Introduction: Semileptonic τ decays



$$M = \frac{G_F}{\sqrt{2}} V_{CKM} \bar{u}(v_\tau) \gamma^\mu (1 - \gamma_5) u(\tau) T_\mu$$

$$T_\mu = \langle \text{Hadrons} | (\mathbf{V}-\mathbf{A})_\mu e^{iS_{\text{QCD}}} | 0 \rangle = \sum_i (\text{Lorentz Structure})^i F_i(Q^2, s_j)$$

Two mesons $h_1(p_1), h_2(p_2)$:

$$J^\mu = N [(p_1 - p_2)^\mu F^V(s) + (p_1 + p_2)^\mu F^S(s)]$$

($T_\mu \sim J_\mu$)

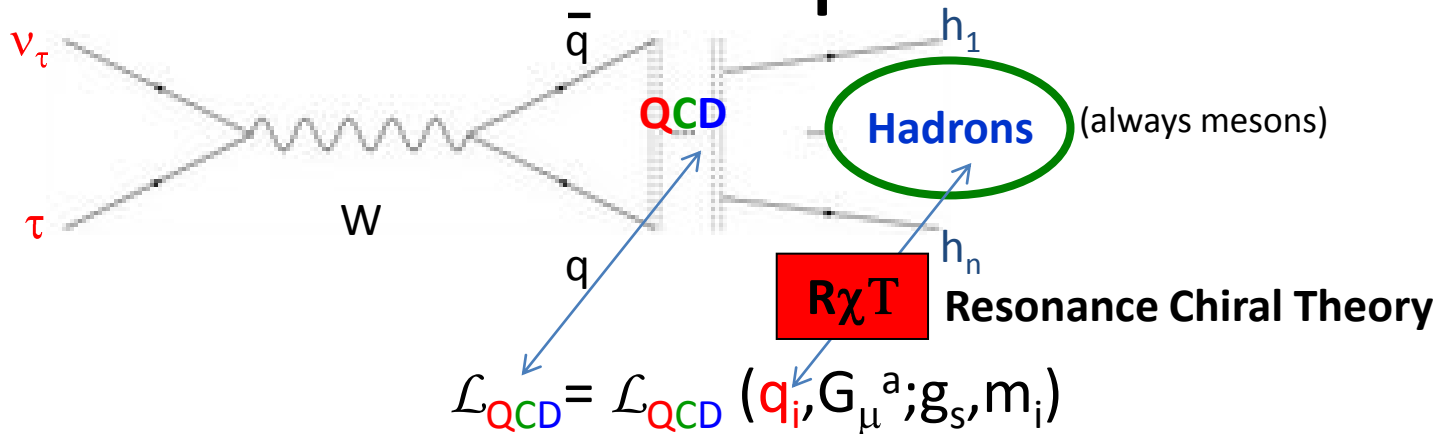
Three mesons $h_1(p_1), h_2(p_2), h_3(p_3)$:

$$J^\mu = N \left\{ T_\nu^\mu [c_1(p_2 - p_3)^\nu F_1 + c_2(p_3 - p_1)^\nu F_2 + c_3(p_1 - p_2)^\nu F_3] + c_4 q^\mu F_4 - \frac{i}{4\pi^2 F^2} c_5 \epsilon^{\mu\nu\rho\sigma} p_1^\nu p_2^\rho p_3^\sigma F_5 \right\}$$

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**

Introduction: Semileptonic τ decays



$$M = \frac{G_F}{\sqrt{2}} V_{CKM} \bar{u}(v_\tau) \gamma^\mu (1 - \gamma_5) u(\tau) T_\mu$$

$$T_\mu = \langle \text{Hadrons} | (\mathbf{V}-\mathbf{A})_\mu e^{iS_{\text{QCD}}} | 0 \rangle = \sum_i (\text{Lorentz Structure})^i F_i(Q^2, s_j)$$

Two mesons $h_1(p_1), h_2(p_2)$:

$$J^\mu = N [(p_1 - p_2)^\mu F^V(s) + (p_1 + p_2)^\mu F^S(s)]$$

($T_\mu \sim J_\mu$)

Form factors

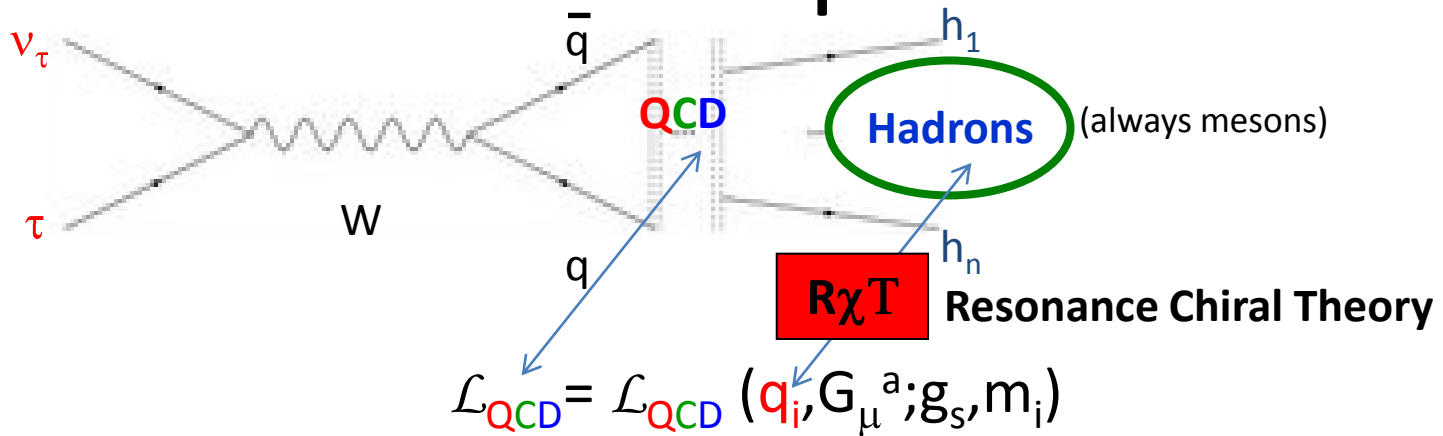
Three mesons $h_1(p_1), h_2(p_2), h_3(p_3)$:

$$J^\mu = N \left\{ T_\nu^\mu [c_1(p_2 - p_3)^\nu F_1 + c_2(p_3 - p_1)^\nu F_2 + c_3(p_1 - p_2)^\nu F_3] + c_4 q^\mu F_4 - \frac{i}{4\pi^2 F^2} c_5 \epsilon^{\mu\nu\rho\sigma} p_1^\nu p_2^\rho p_3^\sigma F_5 \right\}$$

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**

Introduction: Semileptonic τ decays



$$M = \frac{G_F}{\sqrt{2}} V_{CKM} \bar{u}(v_\tau) \gamma^\mu (1 - \gamma_5) u(\tau) T_\mu$$

$$T_\mu = \langle \text{Hadrons} | (\mathbf{V}-\mathbf{A})_\mu e^{iS_{\text{QCD}}} | 0 \rangle = \sum_i (\text{Lorentz Structure})^i F_i(Q^2, s_j)$$

Two mesons $h_1(p_1), h_2(p_2)$:

$$J^\mu = N [(p_1 - p_2)^\mu F^V(s) + (p_1 + p_2)^\mu F^S(s)]$$

$(T_\mu \sim J_\mu)$

Form factors

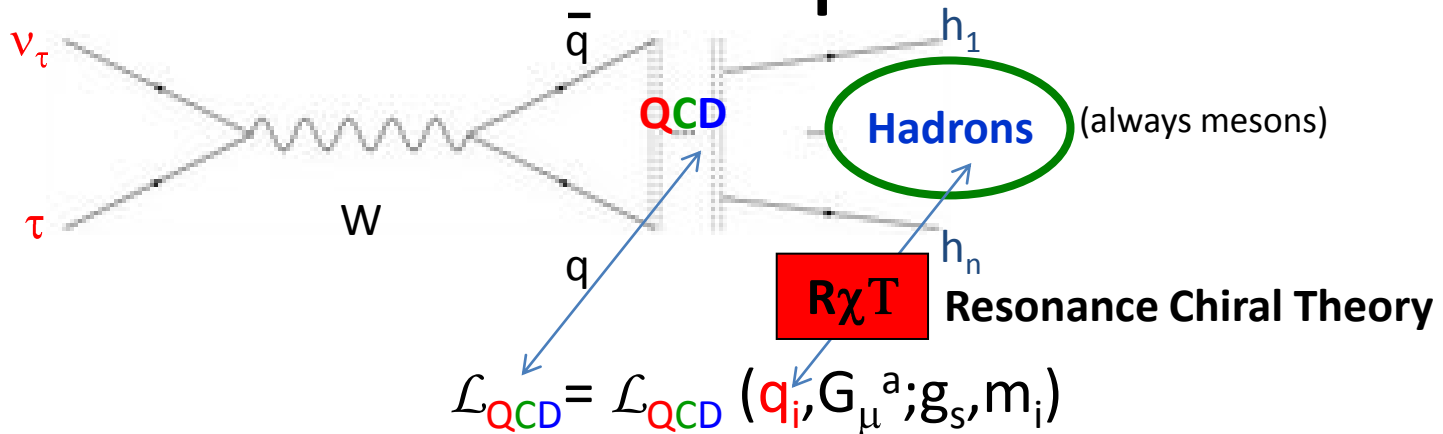
Three mesons $h_1(p_1), h_2(p_2), h_3(p_3)$:

$$J^\mu = N \left\{ T_\nu^\mu [c_1(p_2 - p_3)^\nu F_1 + c_2(p_3 - p_1)^\nu F_2 + c_3(p_1 - p_2)^\nu F_3] + c_4 q^\mu F_4 - \frac{i}{4\pi^2 F^2} c_5 \epsilon^{\mu\nu\rho\sigma} p_1^\nu p_2^\rho p_3^\sigma F_5 \right\}$$

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**

Introduction: Semileptonic τ decays



$$M = \frac{G_F}{\sqrt{2}} V_{CKM} \bar{u}(v_\tau) \gamma^\mu (1 - \gamma_5) u(\tau) T_\mu$$

$$T_\mu = \langle \text{Hadrons} | (\mathbf{V}-\mathbf{A})_\mu e^{iS_{\text{QCD}}} | 0 \rangle = \sum_i (\text{Lorentz Structure})^i F_i(Q^2, s_j)$$

Two mesons $h_1(p_1), h_2(p_2)$:

$$J^\mu = N [(p_1 - p_2)^\mu F^V(s) + (p_1 + p_2)^\mu F^S(s)]$$

($T_\mu \sim J_\mu$)

Form factors

Three mesons $h_1(p_1), h_2(p_2), h_3(p_3)$:

$$J^\mu = N \left\{ T_\nu^\mu [c_1(p_2 - p_3)^\nu F_1 + c_2(p_3 - p_1)^\nu F_2 + c_3(p_1 - p_2)^\nu F_3] + c_4 q^\mu F_4 - \frac{i}{4\pi^2 F^2} c_5 \epsilon^{\mu\nu\rho\sigma} p_1^\nu p_2^\rho p_3^\sigma F_5 \right\}$$

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**

Introduction: Semileptonic τ decays

$$T_\mu = \langle \text{Hadrons} | (\mathbf{V}-\mathbf{A})_\mu e^{iS_{\text{QCD}}} | 0 \rangle = \sum_i (\text{Lorentz Structure})^i F_i(Q^2, s_j)$$

Two mesons $h_1(p_1), h_2(p_2)$: $J^\mu = N [(p_1 - p_2)^\mu F^V(s) + (p_1 + p_2)^\mu F^S(s)]$ $s = (p_1 + p_2)^2$
 $(T_\mu \sim J_\mu)$

Three mesons $h_1(p_1), h_2(p_2), h_3(p_3)$: $J^\mu = N \left\{ T_\nu^\mu [c_1(p_2 - p_3)^\nu F_1 + c_2(p_3 - p_1)^\nu F_2 + c_3(p_1 - p_2)^\nu F_3] + c_4 q^\mu F_4 - \frac{i}{4\pi^2 F^2} c_5 \epsilon^{\mu\nu\rho\sigma} p_1^\nu p_2^\rho p_3^\sigma F_5 \right\}$

$$T_{\mu\nu} = g_{\mu\nu} - q_\mu q_\nu / q^2 \qquad q^\mu = (p_1 + p_2 + p_3)^\mu$$

$$\underline{q^2 = (p_1 + p_2 + p_3)^2}$$

$$\underline{s_1 = (p_2 + p_3)^2}$$

$$\underline{s_2 = (p_1 + p_3)^2}$$

$$s_3 = (p_1 + p_2)^2$$

More mesons $\sim 4\pi$ (Fischer, Wess and Wagner '80; Bondar et. al. '02)

Form factors

Pablo Roig (IFAE)

Theoretical setting: χ PT, Large N_c , $R\chi T$

(Portolés '10)

(Gasser, Leutwyler, '84, '85)

- **QCD** has a well-defined expansion at low-energies that allows to build an **EFT**: χ PT.
- The energy of the hadronic system in semileptonic tau decays is not small enough through all phase space to allow for the low-energy expansion done by χ PT.

(Colangelo, Finkemeier, Urech '96)

- One needs to consider an alternative expansion parameter to extend χ PT to higher energies.

We will take $1/N_c$. ('t Hooft '74, Witten '79)

- LO in the $1/N_c$ -expansion amounts to consider an ∞ **number** of resonances that are strictly **stable**. The Feynman diagrams are given by **tree level** exchanges: local effective interactions only.
- We depart from it in two aspects: - We cut the ∞ spectrum of states in a way that resembles Nature.

- We provide the resonances with a QFT-based off-shell

width within the framework of $R\chi T$. (Gómez-Dumm, Pich, Portolés '00)

(Ecker, Gasser, Pich, De Rafael '89)

(Ecker, Gasser, Leutwyler, Pich, De Rafael '89)

- Finally, one imposes the known **QCD high-energy** behaviour to the **Green functions** or **form factors**.

(Ruiz-Femenía, Pich, Portolés '03)

(Cirigliano, Ecker, Eidemüller, Pich, Portolés '04)

(Cirigliano, Ecker, Eidemüller, Kaiser, Pich, Portolés '05, '06)

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**

$$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$$

Different approaches to deal with the diverse energy regimes

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**

$$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$$

Different approaches to deal with the diverse energy regimes

- For $E < M_\rho \rightarrow \chi$ PT up to $O(p^6)$ (Gasser, Leutwyler'85, Bijns, Colangelo, Talavera '98, Bijns, Talavera'02)

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**

$$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$$

Different approaches to deal with the diverse energy regimes

- For $E < M_\rho \rightarrow \chi$ PT up to $O(p^6)$ (Gasser, Leutwyler '85, Bijmans, Colangelo, Talavera '98, Bijmans, Talavera '02)

(Guerrero, Pich '97)

- For $M_\rho \leq E \leq 1 \text{ GeV} \rightarrow$ Match χ PT results to VMD using an Omnés solution for dispersion relation.

Omnés solution for dispersion relation (Pich, Portolés '01)

Unitarization approach (Trocóniz, Ynduráin '01, Oller, Oser, Palomar '01)

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in TAUOLA

$$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$$

Different approaches to deal with the diverse energy regimes

- For $E < M_\rho \rightarrow \chi$ PT up to $O(p^6)$ (Gasser, Leutwyler '85, Bijmens, Colangelo, Talavera '98, Bijmens, Talavera '02)

(Guerrero, Pich '97)

- For $M_\rho \leq E \leq 1 \text{ GeV} \rightarrow$ Match χ PT results to VMD using an Omnés solution for dispersion relation.

Omnés solution for dispersion relation (Pich, Portolés '01)

Unitarization approach (Trocóniz, Ynduráin '01, Oller, Oser, Palomar '01)

- $1 \text{ GeV} \leq E \leq 2 \text{ GeV} \rightarrow$ Include ρ' through Schwinger-Dyson-like resummation.

(Sanz-Cillero, Pich '03)

Tower of resonances based on dual QCD

(Domínguez '01, Bruch, Khodjamiriam, Kuhn '05)

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**

$$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$$

Different approaches to deal with the diverse energy regimes

- For $E < M_\rho \rightarrow \chi$ PT up to $O(p^6)$ (Gasser, Leutwyler '85, Bijmens, Colangelo, Talavera '98, Bijmens, Talavera '02)

(Guerrero, Pich '97)

- For $M_\rho \leq E \leq 1 \text{ GeV} \rightarrow$ Match χ PT results to VMD using an Omnés solution for dispersion relation.

Omnés solution for dispersion relation (Pich, Portolés '01)

Unitarization approach (Trocóniz, Ynduráin '01, Oller, Oser, Palomar '01)

- $1 \text{ GeV} \leq E \leq 2 \text{ GeV} \rightarrow$ Include ρ' through Schwinger-Dyson-like resummation.

(Sanz-Cillero, Pich '03)

Tower of resonances based on dual QCD

(Domínguez '01, Bruch, Khodjamiriam, Kuhn '05)

This will be our starting point

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in TAUOLA

$$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$$

(Guerrero, Pich '97)

Match χ PT results to VMD using an Omnés solution for dispersion relation

$$\langle \pi^0 \pi^- | \bar{d} \gamma^\mu u | \emptyset \rangle = \sqrt{2} F(s) (p_{\pi^-} - p_{\pi^0})^\mu \quad s \equiv q^2 \equiv (p_{\pi^-} + p_{\pi^0})^2$$

Pablo Roig (IFAE)

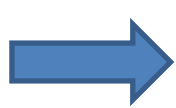
$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**

$$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$$

(Guerrero, Pich '97)

Match χ PT results to VMD using an Omnés solution for dispersion relation

$$\langle \pi^0 \pi^- | \bar{d} \gamma^\mu u | \emptyset \rangle = \sqrt{2} F(s) (p_{\pi^-} - p_{\pi^0})^\mu \quad s \equiv q^2 \equiv (p_{\pi^-} + p_{\pi^0})^2$$



$$F(s)_{\text{ChPT}}^{\text{O}(p^4)} = 1 + \frac{2L_9^r(\mu)}{f_\pi^2} s - \frac{s}{96\pi^2 f_\pi^2} \left[A(m_\pi^2/s, m_\pi^2/\mu^2) + \frac{1}{2} A(m_K^2/s, m_K^2/\mu^2) \right]$$

Pablo Roig (IFAE)

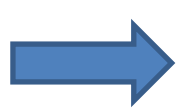
$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**

$$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$$

(Guerrero, Pich '97)

Match χ PT results to VMD using an Omnés solution for dispersion relation

$$\langle \pi^0 \pi^- | \bar{d} \gamma^\mu u | \emptyset \rangle = \sqrt{2} F(s) (p_{\pi^-} - p_{\pi^0})^\mu \quad s \equiv q^2 \equiv (p_{\pi^-} + p_{\pi^0})^2$$



$$F(s)_{\text{O}(p^4)}^{\text{ChPT}} = 1 + \frac{2L_9^r(\mu)}{f_\pi^2} s - \frac{s}{96\pi^2 f_\pi^2} \left[A(m_\pi^2/s, m_\pi^2/\mu^2) + \frac{1}{2} A(m_K^2/s, m_K^2/\mu^2) \right]$$

$$A(m_P^2/s, m_P^2/\mu^2) = \ln(m_P^2/\mu^2) + \frac{8m_P^2}{s} - \frac{5}{3} + \sigma_P^3 \ln\left(\frac{\sigma_P + 1}{\sigma_P - 1}\right) \quad \sigma_P \equiv \sqrt{1 - 4m_P^2/s}$$

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in TAUOLA

$$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$$

(Guerrero, Pich '97)

Match χ PT results to VMD using an Omnés solution for dispersion relation

$$\langle \pi^0 \pi^- | \bar{d} \gamma^\mu u | \emptyset \rangle = \sqrt{2} F(s) (p_{\pi^-} - p_{\pi^0})^\mu \quad s \equiv q^2 \equiv (p_{\pi^-} + p_{\pi^0})^2$$

$$\rightarrow F(s)_{\text{ChPT}}^{\mathcal{O}(p^4)} = 1 + \frac{2L_9^r(\mu)}{f_\pi^2} s - \frac{s}{96\pi^2 f_\pi^2} \left[A(m_\pi^2/s, m_\pi^2/\mu^2) + \frac{1}{2} A(m_K^2/s, m_K^2/\mu^2) \right]$$

$$A(m_P^2/s, m_P^2/\mu^2) = \ln(m_P^2/\mu^2) + \frac{8m_P^2}{s} - \frac{5}{3} + \sigma_P^3 \ln\left(\frac{\sigma_P + 1}{\sigma_P - 1}\right) \quad \sigma_P \equiv \sqrt{1 - 4m_P^2/s}$$

$$\rightarrow F(s)^V = 1 + \frac{F_V G_V}{f_\pi^2} \frac{s}{M_\rho^2 - s}$$

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**

$$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$$

(Guerrero, Pich '97)

Match χ PT results to VMD using an Omnés solution for dispersion relation

$$\langle \pi^0 \pi^- | \bar{d} \gamma^\mu u | 0 \rangle = \sqrt{2} F(s) (p_{\pi^-} - p_{\pi^0})^\mu \quad s \equiv q^2 \equiv (p_{\pi^-} + p_{\pi^0})^2$$

$$\rightarrow F(s)_{\text{ChPT}}^{\text{O}(p^4)} = 1 + \frac{2L_9^r(\mu)}{f_\pi^2} s - \frac{s}{96\pi^2 f_\pi^2} \left[A(m_\pi^2/s, m_\pi^2/\mu^2) + \frac{1}{2} A(m_K^2/s, m_K^2/\mu^2) \right]$$

$$A(m_P^2/s, m_P^2/\mu^2) = \ln(m_P^2/\mu^2) + \frac{8m_P^2}{s} - \frac{5}{3} + \sigma_P^3 \ln\left(\frac{\sigma_P + 1}{\sigma_P - 1}\right) \quad \sigma_P \equiv \sqrt{1 - 4m_P^2/s}$$

$$\rightarrow F(s)^V = 1 + \frac{F_V G_V}{f_\pi^2} \frac{s}{M_\rho^2 - s} \quad F(s) \rightarrow 0, \text{ for } s \rightarrow \infty \Rightarrow F_V G_V / f_\pi^2 = 1$$

$$F(s)^{\text{VMD}} = \frac{M_\rho^2}{M_\rho^2 - s}$$

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**

$$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$$

(Guerrero, Pich '97)

Match χ PT results to VMD using an Omnés solution for dispersion relation

$$\langle \pi^0 \pi^- | \bar{d} \gamma^\mu u | 0 \rangle = \sqrt{2} F(s) (p_{\pi^-} - p_{\pi^0})^\mu \quad s \equiv q^2 \equiv (p_{\pi^-} + p_{\pi^0})^2$$

$$\rightarrow F(s)_{\text{O}(p^4)}^{\text{ChPT}} = 1 + \frac{2L_9^r(\mu)}{f_\pi^2} s - \frac{s}{96\pi^2 f_\pi^2} \left[A(m_\pi^2/s, m_\pi^2/\mu^2) + \frac{1}{2} A(m_K^2/s, m_K^2/\mu^2) \right]$$

$$A(m_P^2/s, m_P^2/\mu^2) = \ln(m_P^2/\mu^2) + \frac{8m_P^2}{s} - \frac{5}{3} + \sigma_P^3 \ln\left(\frac{\sigma_P + 1}{\sigma_P - 1}\right) \quad \sigma_P \equiv \sqrt{1 - 4m_P^2/s}$$

$$\rightarrow F(s)^V = 1 + \frac{F_V G_V}{f_\pi^2} \frac{s}{M_\rho^2 - s} \quad F(s) \rightarrow 0, \text{ for } s \rightarrow \infty \Rightarrow F_V G_V / f_\pi^2 = 1$$

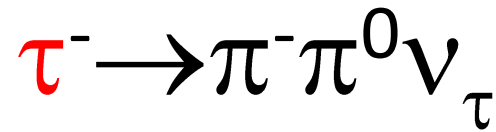
$$F(s)^{\text{VMD}} = \frac{M_\rho^2}{M_\rho^2 - s}$$

ChPT+VMD

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s} - \frac{s}{96\pi^2 f_\pi^2} \left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2} A(m_K^2/s, m_K^2/M_\rho^2) \right]$$

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in TAUOLA



ChPT+VMD (Guerrero, Pich '97)

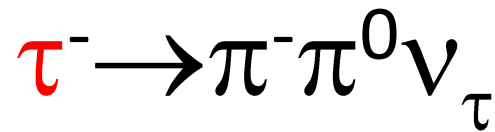
$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s} - \frac{s}{96\pi^2 f_\pi^2} \left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2} A(m_K^2/s, m_K^2/M_\rho^2) \right]$$

Unitarity+Analyticity (Omnés, '58)



Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**



ChPT+VMD (Guerrero, Pich '97)

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s} - \frac{s}{96\pi^2 f_\pi^2} \left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2} A(m_K^2/s, m_K^2/M_\rho^2) \right]$$

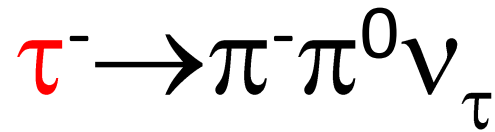
Unitarity+Analyticity (Omnés, '58)

O(p²) result for $\delta_1^1(s)$

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s} \exp \left\{ \frac{-s}{96\pi^2 f_\pi^2} \left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2} A(m_K^2/s, m_K^2/M_\rho^2) \right] \right\}$$

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in TAUOLA



ChPT+VMD (Guerrero, Pich '97)

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s} - \frac{s}{96\pi^2 f_\pi^2} \left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2} A(m_K^2/s, m_K^2/M_\rho^2) \right]$$

Unitarity+Analyticity (Omnés, '58)

O(p²) result for $\delta_1^1(s)$

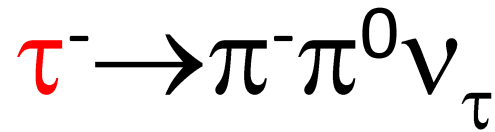
$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s} \exp \left\{ \frac{-s}{96\pi^2 f_\pi^2} \left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2} A(m_K^2/s, m_K^2/M_\rho^2) \right] \right\}$$

(Guerrero, Pich '97) $\Gamma_\rho(s) = \frac{M_\rho s}{96\pi f_\pi^2} \left\{ \theta(s - 4m_\pi^2) \sigma_\pi^3 + \frac{1}{2} \theta(s - 4m_K^2) \sigma_K^3 \right\}$

(Gómez-Dumm, Pich, Portolés '00) $= -\frac{M_\rho s}{96\pi^2 f_\pi^2} \text{Im} \left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2} A(m_K^2/s, m_K^2/M_\rho^2) \right]$

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in TAUOLA



ChPT+VMD (Guerrero, Pich '97)

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s} - \frac{s}{96\pi^2 f_\pi^2} \left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2} A(m_K^2/s, m_K^2/M_\rho^2) \right]$$

Unitarity+Analyticity (Omnés, '58)

O(p²) result for $\delta_1^1(s)$

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s} \exp \left\{ \frac{-s}{96\pi^2 f_\pi^2} \left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2} A(m_K^2/s, m_K^2/M_\rho^2) \right] \right\}$$

$$\text{(Guerrero, Pich '97)} \Gamma_\rho(s) = \frac{M_\rho s}{96\pi f_\pi^2} \left\{ \theta(s - 4m_\pi^2) \sigma_\pi^3 + \frac{1}{2} \theta(s - 4m_K^2) \sigma_K^3 \right\}$$

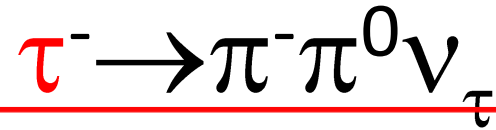
$$\text{(Gómez-Dumm, Pich, Portolés '00)} = -\frac{M_\rho s}{96\pi^2 f_\pi^2} \text{Im} \left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2} A(m_K^2/s, m_K^2/M_\rho^2) \right]$$

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp \left\{ \frac{-s}{96\pi^2 f_\pi^2} \left[\text{Re} A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2} \text{Re} A(m_K^2/s, m_K^2/M_\rho^2) \right] \right\}$$

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in TAUOLA

Our starting point



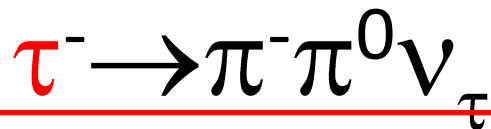
(Guerrero, Pich '97)

Match χ PT results to VMD using an Omnés solution for dispersion relation

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp \left\{ \frac{-s}{96\pi^2 f_\pi^2} \left[\text{Re}A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2} \text{Re}A(m_K^2/s, m_K^2/M_\rho^2) \right] \right\}$$

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**



Our starting point

(Guerrero, Pich '97)

Match χ PT results to VMD using an Omnés solution for dispersion relation

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp \left\{ \frac{-s}{96\pi^2 f_\pi^2} \left[\text{Re}A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2} \text{Re}A(m_K^2/s, m_K^2/M_\rho^2) \right] \right\}$$

- χ PT up to $O(p^4)$ and leading $O(p^6)$ contributions
- Right fall-off at high energies

- SU(2)
- Analyticity and unitarity constraints

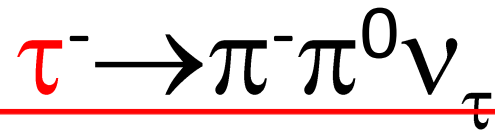


Idea: Follow the approach of (Jamin, Pich, Portolés '06) including excited resonances while retaining (some of) these nice properties

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in TAUOLA

Our starting point



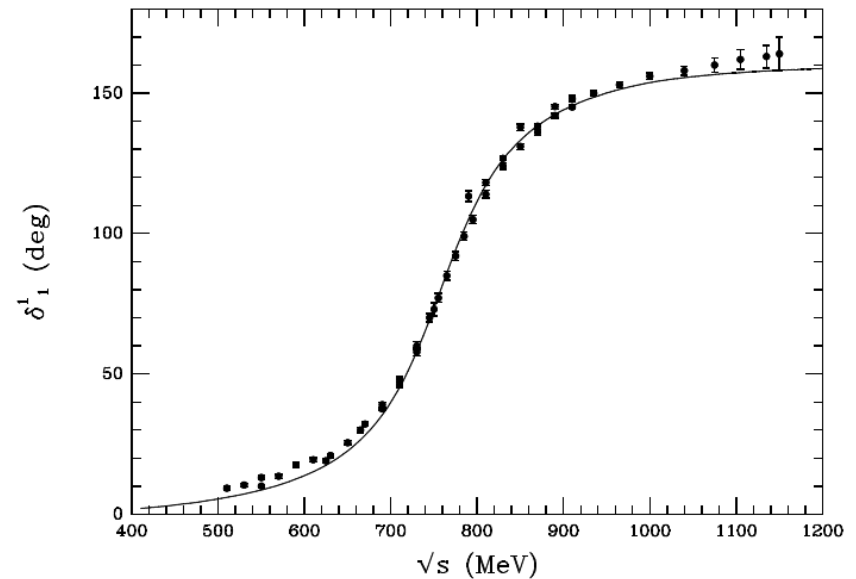
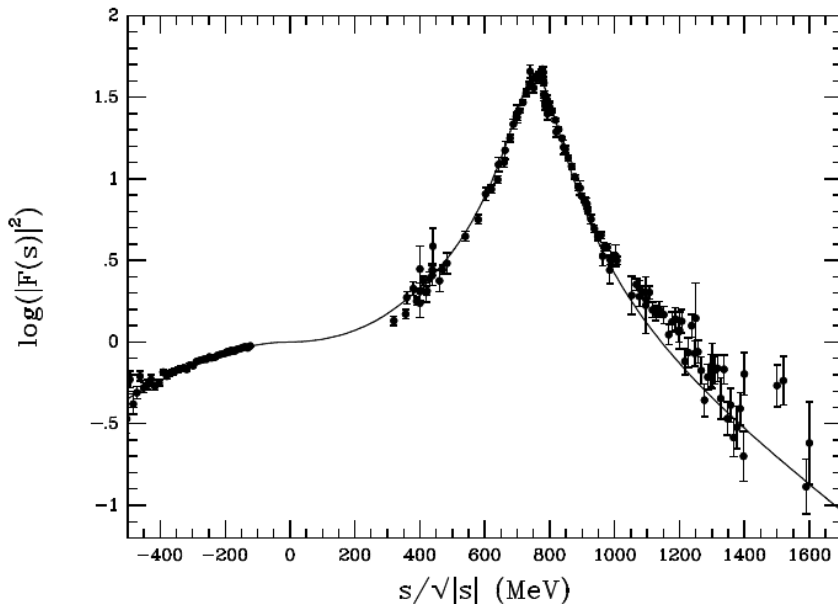
(Guerrero, Pich '97)

Match χ PT results to VMD using an Omnés solution for dispersion relation

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left\{ \frac{-s}{96\pi^2 f_\pi^2} \left[\text{Re}A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}\text{Re}A(m_K^2/s, m_K^2/M_\rho^2) \right] \right\}$$

- χ PT up to $O(p^4)$ and leading $O(p^6)$ contributions
- Right fall-off at high energies

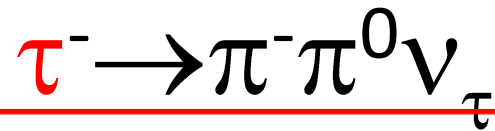
- SU(2)
- Analyticity and unitarity constraints



Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in TAUOLA

Our starting point



(Guerrero, Pich '97)

Match χ PT results to VMD using an Omnés solution for dispersion relation

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp \left\{ \frac{-s}{96\pi^2 f_\pi^2} \left[\Re A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2} \Re A(m_K^2/s, m_K^2/M_\rho^2) \right] \right\}$$

Our formula $F_V^-(s)$

$$= \frac{M_\rho^2 + s(\gamma e^{i\phi_1} + \delta e^{i\phi_2})}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp \left[\frac{-s}{96\pi^2 F_\pi^2} (\Re A_\pi(s) + \Re A_K(s)/2) \right]$$

$$- \frac{\gamma s e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'} \Gamma_{\rho'}(s)} \exp \left[\frac{-s \Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)} \Re A_\pi(s) \right]$$

$$- \frac{\delta s e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''} \Gamma_{\rho''}(s)} \exp \left[\frac{-s \Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)} \Re A_\pi(s) \right].$$

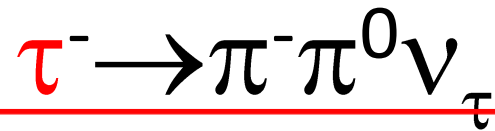
- χ PT up to $O(p^4)$ and leading $O(p^6)$ contributions
- Right fall-off at high energies
- SU(2)

- Analyticity and unitarity constraints
- (Phenomenological) contribution of $\rho' + \rho''$

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**

Our starting point



(Guerrero, Pich '97)

Match χ PT results to VMD using an Omnés solution for dispersion relation

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp \left\{ \frac{-s}{96\pi^2 f_\pi^2} \left[\text{Re}A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2} \text{Re}A(m_K^2/s, m_K^2/M_\rho^2) \right] \right\}$$

Our formula $F_V^-(s)$

$$= \frac{M_\rho^2 + s(\gamma e^{i\phi_1} + \delta e^{i\phi_2})}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp \left[\frac{-s}{96\pi^2 F_\pi^2} (\Re A_\pi(s) + \Re A_K(s)/2) \right]$$

$$- \frac{\gamma s e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'} \Gamma_{\rho'}(s)} \exp \left[\frac{-s \Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)} \Re A_\pi(s) \right]$$

$$- \frac{\delta s e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''} \Gamma_{\rho''}(s)} \exp \left[\frac{-s \Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)} \Re A_\pi(s) \right].$$

- χ PT up to $O(p^4)$ and leading $O(p^6)$ contributions
- Right fall-off at high energies
- SU(2)

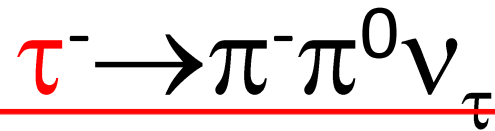
- Analyticity and unitarity constraints
- (Phenomenological) contribution of $\rho' + \rho''$

This is what is included in **TAUOLA** right now

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**

Our starting point



(Guerrero, Pich '97)

Match χ PT results to VMD using an Omnés solution for dispersion relation

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp \left\{ \frac{-s}{96\pi^2 f_\pi^2} \left[\text{Re}A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2} \text{Re}A(m_K^2/s, m_K^2/M_\rho^2) \right] \right\}$$

Our formula $F_V^-(s)$

$$= \frac{M_\rho^2 + s(\gamma e^{i\phi_1} + \delta e^{i\phi_2})}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp \left[\frac{-s}{96\pi^2 F_\pi^2} (\Re A_\pi(s) + \Re A_K(s)/2) \right]$$

$$- \frac{\gamma s e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'} \Gamma_{\rho'}(s)} \exp \left[\frac{-s \Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)} \Re A_\pi(s) \right]$$

$$- \frac{\delta s e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''} \Gamma_{\rho''}(s)} \exp \left[\frac{-s \Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)} \Re A_\pi(s) \right].$$

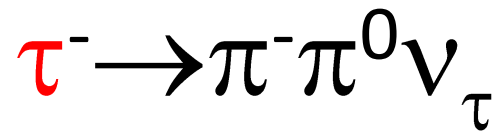
- χ PT up to $O(p^4)$ and leading $O(p^6)$ contributions
- Right fall-off at high energies
- SU(2)

- Analyticity and unitarity constraints
- (Phenomenological) contribution of $\rho' + \rho''$

This is what is included in **TAUOLA** right now

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**



Our formula $F_V^-(s)$ =

$$= \frac{M_\rho^2 + s(\gamma e^{i\phi_1} + \delta e^{i\phi_2})}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp \left[\frac{-s}{96\pi^2 F_\pi^2} (\Re A_\pi(s) + \Re A_K(s)/2) \right]$$

$$- \frac{\gamma s e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'} \Gamma_{\rho'}(s)} \exp \left[\frac{-s \Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)} \Re A_\pi(s) \right]$$

$$- \frac{\delta s e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''} \Gamma_{\rho''}(s)} \exp \left[\frac{-s \Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)} \Re A_\pi(s) \right].$$

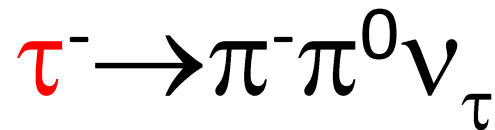
SU(2) breaking: $m_{\pi^+} \neq m_{\pi^0}$, $m_{K^+} \neq m_{K^0}$ in resonance widths and loop functions

$$\Gamma_\rho(s) = \frac{s M_\rho}{96 \pi F_\pi^2} \left[\theta(x - thr_\pi) \lambda^{3/2} \left(1, \frac{m_{\pi^+}^2}{s}, \frac{m_{\pi^0}^2}{s} \right) + \frac{1}{2} \theta(x - thr_K) \lambda^{3/2} \left(1, \frac{m_{K^+}^2}{s}, \frac{m_{K^0}^2}{s} \right) \right]$$

$$A_\pi(s) \longrightarrow A_{PQ}(s) = -\frac{192 \pi^2 [s M_{PQ}(s) - L_{PQ}(s)]}{s}$$

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**



Our formula $F_V^-(s)$ =

$$\frac{M_\rho^2 + s(\gamma e^{i\phi_1} + \delta e^{i\phi_2})}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp \left[\frac{-s}{96\pi^2 F_\pi^2} (\Re A_\pi(s) + \Re A_K(s)/2) \right]$$

$$- \frac{\gamma s e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'} \Gamma_{\rho'}(s)} \exp \left[\frac{-s \Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)} \Re A_\pi(s) \right]$$

$$- \frac{\delta s e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''} \Gamma_{\rho''}(s)} \exp \left[\frac{-s \Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)} \Re A_\pi(s) \right].$$

SU(2) breaking: $m_{\pi^+} \neq m_{\pi^0}$, $m_{K^+} \neq m_{K^0}$ in resonance widths and loop functions

electromagnetic corrections

(Cirigliano, Ecker and Neufeld '01,'02, Flores-Báez, Flores-Tlalpa, López-Castro and Toledo '06)

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**



Our formula $F_V^-(s)$ =

$$\frac{M_\rho^2 + s(\gamma e^{i\phi_1} + \delta e^{i\phi_2})}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp \left[\frac{-s}{96\pi^2 F_\pi^2} (\Re A_\pi(s) + \Re A_K(s)/2) \right]$$

$$- \frac{\gamma s e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'} \Gamma_{\rho'}(s)} \exp \left[\frac{-s \Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)} \Re A_\pi(s) \right]$$

$$- \frac{\delta s e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''} \Gamma_{\rho''}(s)} \exp \left[\frac{-s \Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)} \Re A_\pi(s) \right].$$

SU(2) breaking: $m_{\pi^+} \neq m_{\pi^0}$, $m_{K^+} \neq m_{K^0}$ in resonance widths and loop functions

electromagnetic corrections

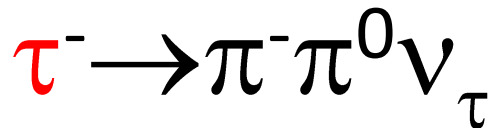
(Cirigliano, Ecker and Neufeld '01,'02, Flores-Báez, Flores-Tlalpa, López-Castro and Toledo '06)

$$\frac{M_\rho^2 + s(\gamma e^{i\phi_1} + \delta e^{i\phi_2})}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp \left[\frac{-s}{96\pi^2 F_\pi^2} (\Re A_{\pi^- \pi^0}(s) + \Re A_{K^- K^0}(s)/2) \right] + f_{\text{local}}^{\text{elm}}$$

$$f_{\text{local}}^{\text{elm}} = \frac{\alpha}{4\pi} \left[-\frac{3}{2} - \frac{1}{2} \text{Ln} \frac{M_\tau^2}{\mu^2} - \text{Ln} \frac{m_\pi^2}{\mu^2} + 2 \text{Ln} \frac{M_\tau^2}{M_\rho^2} - (4\pi)^2 \left(-2K_{12}^r(\mu) + \frac{2}{3} X_1 + \frac{1}{6} \tilde{X}_6^r(\mu) \right) \right]$$

Pablo Roig (IFAE)

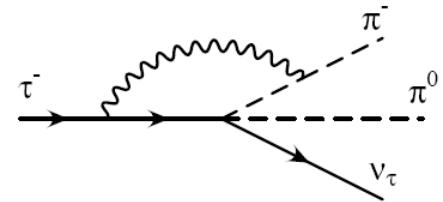
$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**



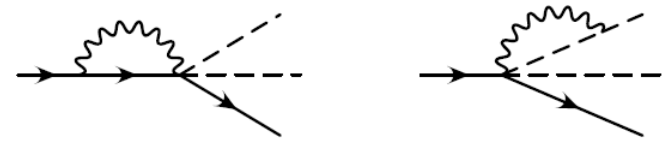
(Cirigliano, Ecker and Neufeld '01)

$$\frac{d\Gamma(\tau^- \rightarrow \pi^0 \pi^- \nu_\tau)}{dt} = \frac{\Gamma_e^{(0)} S_{EW} |V_{ud}|^2}{2m_\tau^2} \beta_{\pi^0 \pi^-}(t) \left(1 - \frac{t}{m_\tau^2}\right)^2 \left\{ |f_+(t)|^2 \left[\left(1 + \frac{2t}{m_\tau^2}\right) \beta_{\pi^0 \pi^-}^2(t) + \frac{3\Delta_\pi^2}{t^2} \right] + 3|f_-(t)|^2 - 6\text{Re} [f_+^*(t) f_-(t)] \frac{\Delta_\pi}{t} \right\} G_{EM}(t)$$

(t corresponds to s before)

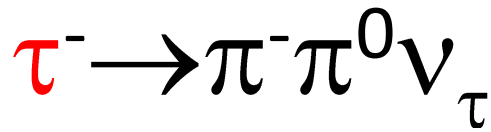


(some of the diagrams contributing to $G_{EM}(s)$)



Pablo Roig (IFAE)

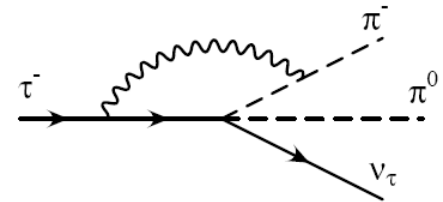
$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**



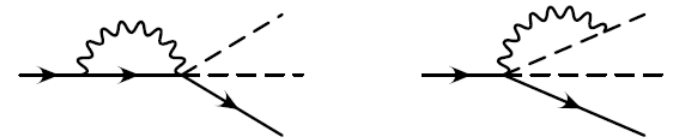
(Cirigliano, Ecker and Neufeld '01)

$$\frac{d\Gamma(\tau^- \rightarrow \pi^0 \pi^- \nu_\tau)}{dt} = \frac{\Gamma_e^{(0)} S_{EW} |V_{ud}|^2}{2m_\tau^2} \beta_{\pi^0 \pi^-}(t) \left(1 - \frac{t}{m_\tau^2}\right)^2 \left\{ |f_+(t)|^2 \left[\left(1 + \frac{2t}{m_\tau^2}\right) \beta_{\pi^0 \pi^-}^2(t) + \frac{3\Delta_\pi^2}{t^2} \right] + 3|f_-(t)|^2 - 6\text{Re} [f_+^*(t) f_-(t)] \frac{\Delta_\pi}{t} \right\} G_{EM}(t)$$

(t corresponds to s before)



(some of the diagrams contributing to $G_{EM}(s)$)

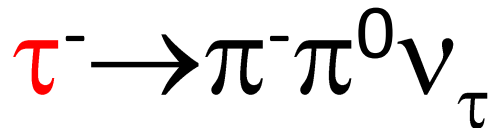


In Belle fits to their invariant mass distribution (Belle '08), SU(2) breaking is incorporated through different masses for the neutral and charged scalars.

$G_{EM}(s)$ is not taken into account at this step, but for the extraction of the $\pi\pi$ part to the hadronic contribution to a_μ .

Pablo Roig (IFAE)

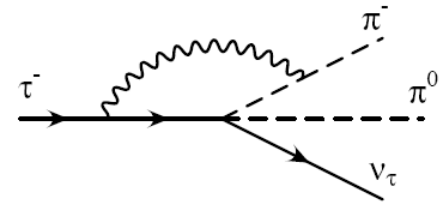
$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**



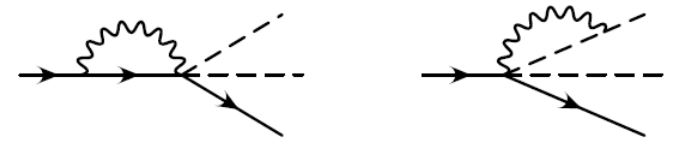
(Cirigliano, Ecker and Neufeld '01)

$$\frac{d\Gamma(\tau^- \rightarrow \pi^0 \pi^- \nu_\tau)}{dt} = \frac{\Gamma_e^{(0)} S_{EW} |V_{ud}|^2}{2m_\tau^2} \beta_{\pi^0 \pi^-}(t) \left(1 - \frac{t}{m_\tau^2}\right)^2 \left\{ |f_+(t)|^2 \left[\left(1 + \frac{2t}{m_\tau^2}\right) \beta_{\pi^0 \pi^-}^2(t) + \frac{3\Delta_\pi^2}{t^2} \right] + 3|f_-(t)|^2 - 6\text{Re} [f_+^*(t) f_-(t)] \frac{\Delta_\pi}{t} \right\} G_{EM}(t)$$

(t corresponds to s before)



(some of the diagrams contributing to $G_{EM}(s)$)

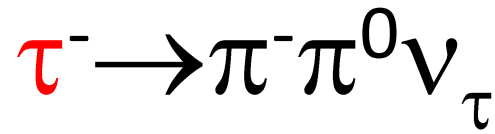


Three types of fits can be considered:

1. SU(2) case, as it is now in TAUOLA
2. SU(2) breaking through $m_{\pi^+} \neq m_{\pi^0}$, $m_{K^+} \neq m_{K^0}$
3. SU(2) breaking including f_{local}^{elm} and $G_{EM}(s)$

Pablo Roig (IFAE)

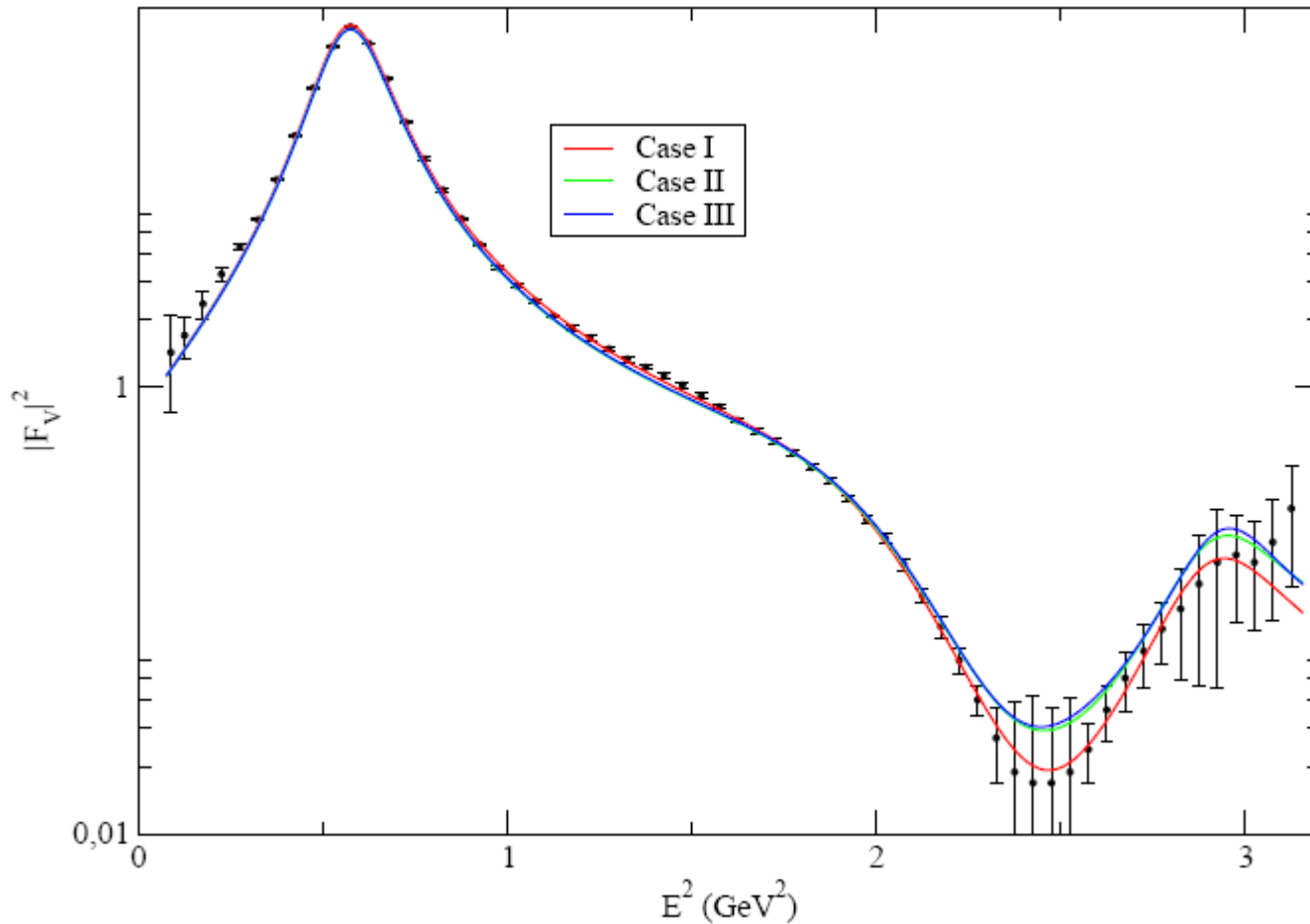
$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in TAUOLA



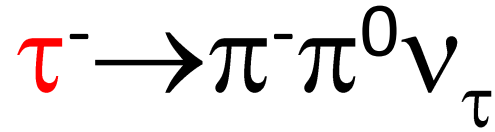
Three types of fits can be considered:

1. SU(2) case, as it is now in TAUOLA
2. SU(2) breaking through $m_{\pi^+} \neq m_{\pi^0}$, $m_{K^+} \neq m_{K^0}$
3. SU(2) breaking including $f_{\text{local}}^{\text{elm}}$ and $G_{\text{FM}}(s)$

Our fits



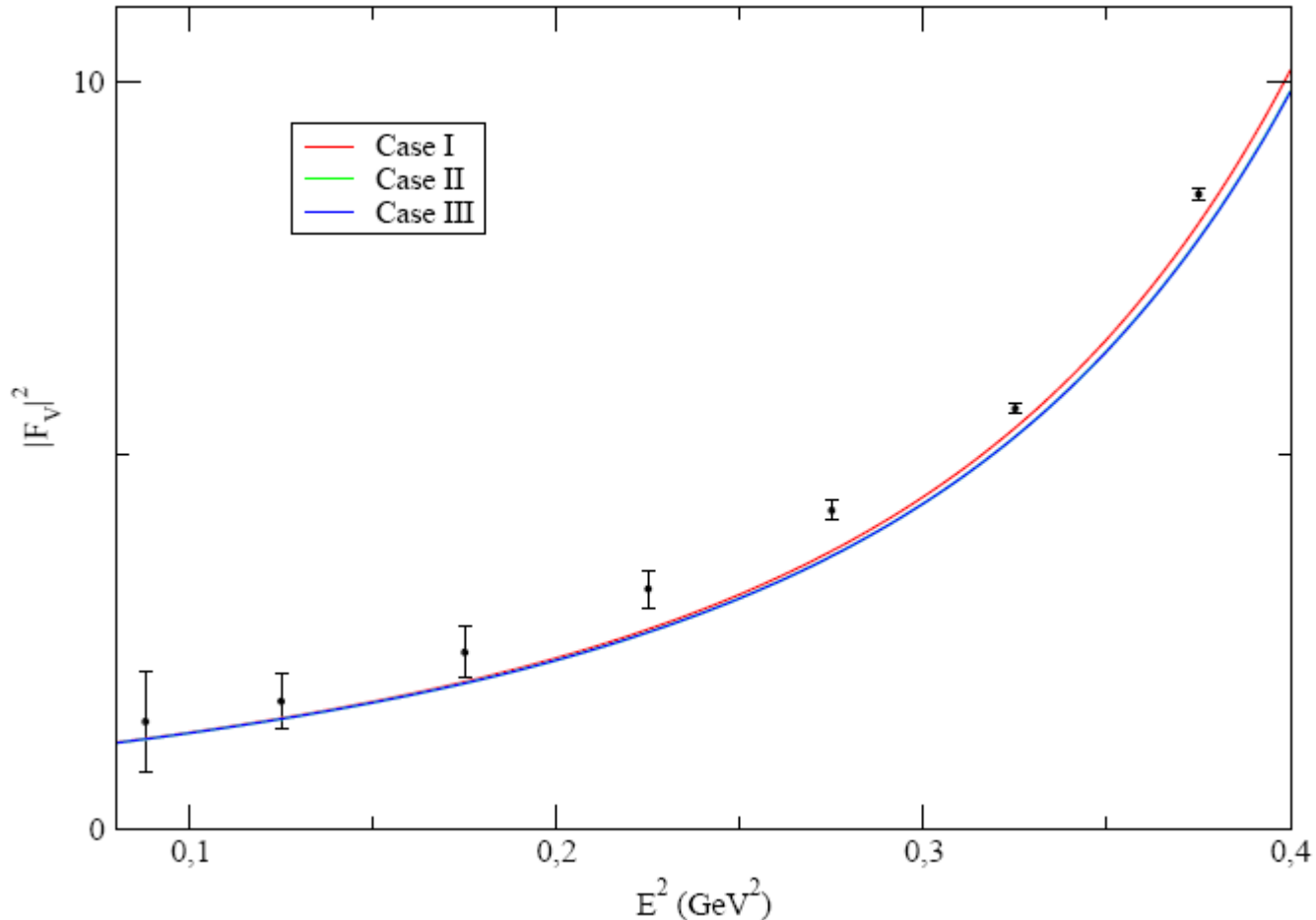
Pablo Roig (IFAE)



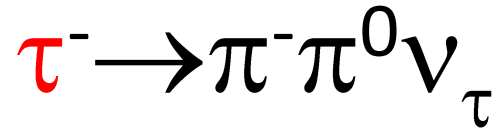
Three types of fits can be considered:

1. SU(2) case, as it is now in TAUOLA
2. SU(2) breaking through $m_{\pi^+} \neq m_{\pi^0}$, $m_{K^+} \neq m_{K^0}$
3. SU(2) breaking including $f^{\text{elm}}_{\text{local}}$ and $G_{\text{FM}}(s)$

Our fits



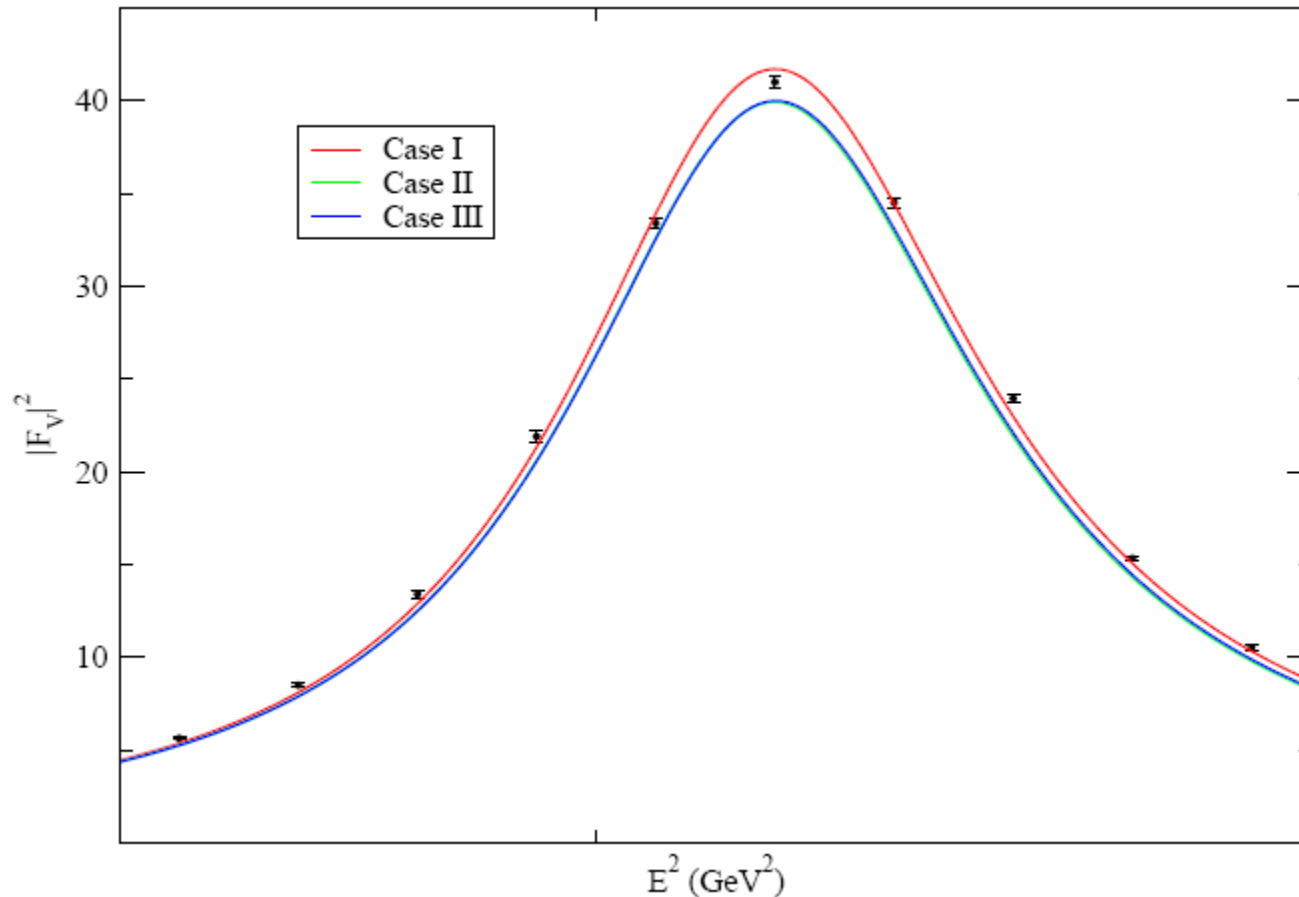
Pablo Roig (IFAE)



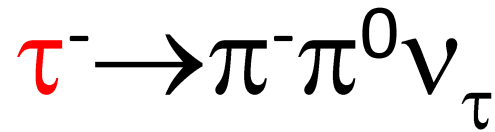
Three types of fits can be considered:

1. SU(2) case, as it is now in TAUOLA
2. SU(2) breaking through $m_{\pi^+} \neq m_{\pi^0}$, $m_{K^+} \neq m_{K^0}$
3. SU(2) breaking including $f_{\text{local}}^{\text{elm}}$ and $G_{\text{EM}}(s)$

Our fits



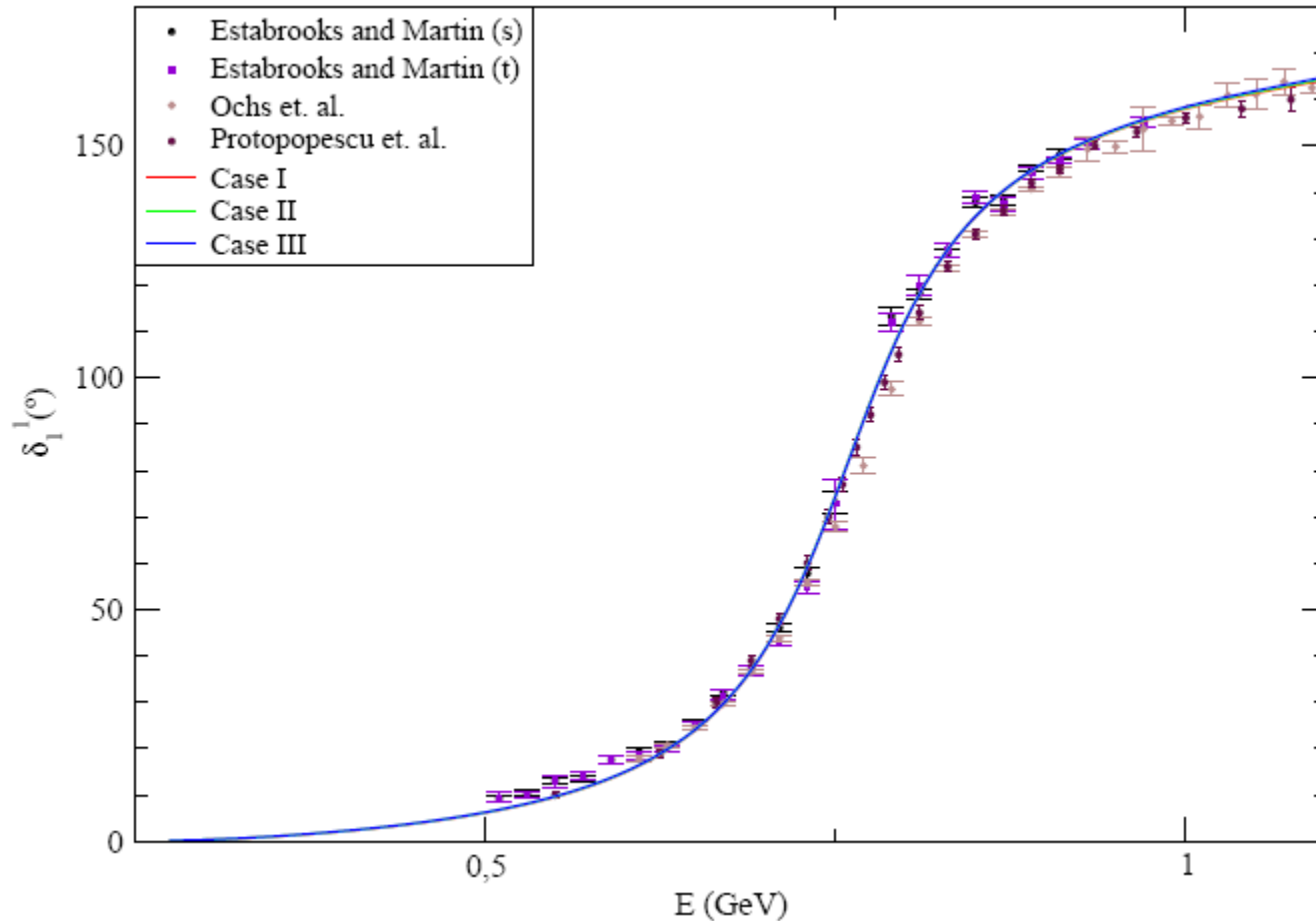
Pablo Roig (IFAE)



Three types of fits can be considered:

1. SU(2) case, as it is now in TAUOLA
2. SU(2) breaking through $m_{\pi^+} \neq m_{\pi^0}$, $m_{K^+} \neq m_{K^0}$
3. SU(2) breaking including $f^{\text{elm}}_{\text{local}}$ and $G_{\text{EM}}(s)$

Our fits



Pablo Roig (IFAE)

Other two meson τ decay channels: $(K\pi)^-$, K^-K^0

In principle, we will follow the same philosophy as in the two pion decay (no $R\chi T$ coupling free)

$$F_{KK}^V(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp \left\{ \frac{-s}{96\pi^2 F^2} \left[\text{Re}A_\pi(s) + \frac{1}{2}\text{Re}A_K(s) \right] \right\}$$

(Guerrero, Pich '97, Arganda, Herrero, Portolés '08)

(Boito, Escribano, Jamin '10)

$$\langle \pi^-(p) | \bar{s} \gamma^\mu u | K^0(k) \rangle = \left[(k+p)^\mu - \frac{m_K^2 - m_\pi^2}{q^2} (k-p)^\mu \right] F_+(q^2) + \frac{m_K^2 - m_\pi^2}{q^2} (k-p)^\mu F_0(q^2)$$

$$\tilde{F}_{+,0}(q^2) \equiv \frac{F_{+,0}(q^2)}{F_+(0)}$$

$$\tilde{F}_+^{K\pi}(s) = \frac{m_{K^*}^2 - \kappa_{K^*} \bar{A}_{K\pi}(0) + \gamma s}{D(m_{K^*}, \gamma_{K^*})} - \frac{\gamma s}{D(m_{K^{*'}}, \gamma_{K^{*'}})}$$

$$D(m_n, \gamma_n) = m_n^2 - s - \kappa_n \text{Re} \bar{A}_{K\pi}(s) - i m_n \gamma_n(s),$$

$$\gamma_{K^*}(s) = \gamma_{K^*} \frac{s}{m_{K^*}^2} \frac{\sigma_{K\pi}^3(s)}{\sigma_{K\pi}^3(m_{K^*}^2)}, \quad \kappa_{K^*} = \frac{192\pi F F_K}{\sigma_{K\pi}(m_{K^*}^2)^3} \frac{\gamma_{K^*}}{m_{K^*}},$$

$$\sigma_{K\pi}(s) = \sqrt{\left(1 - \frac{(m_K + m_\pi)^2}{s}\right) \left(1 - \frac{(m_K - m_\pi)^2}{s}\right)} \theta(s - (m_K + m_\pi)^2)$$

$$F_+^{K\pi}(0) = \frac{m_{K^*}^2}{m_{K^*}^2 - \kappa \bar{A}_{K\pi}(0)}$$

Other approaches: (Jamin, Pich, Portolés '06, '08)

(Boito, Escribano, Jamin '06)

(Bernard, Boito, Passemar '11)

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**

Other two meson τ decay channels: $(K\pi)^-$, K^-K^0

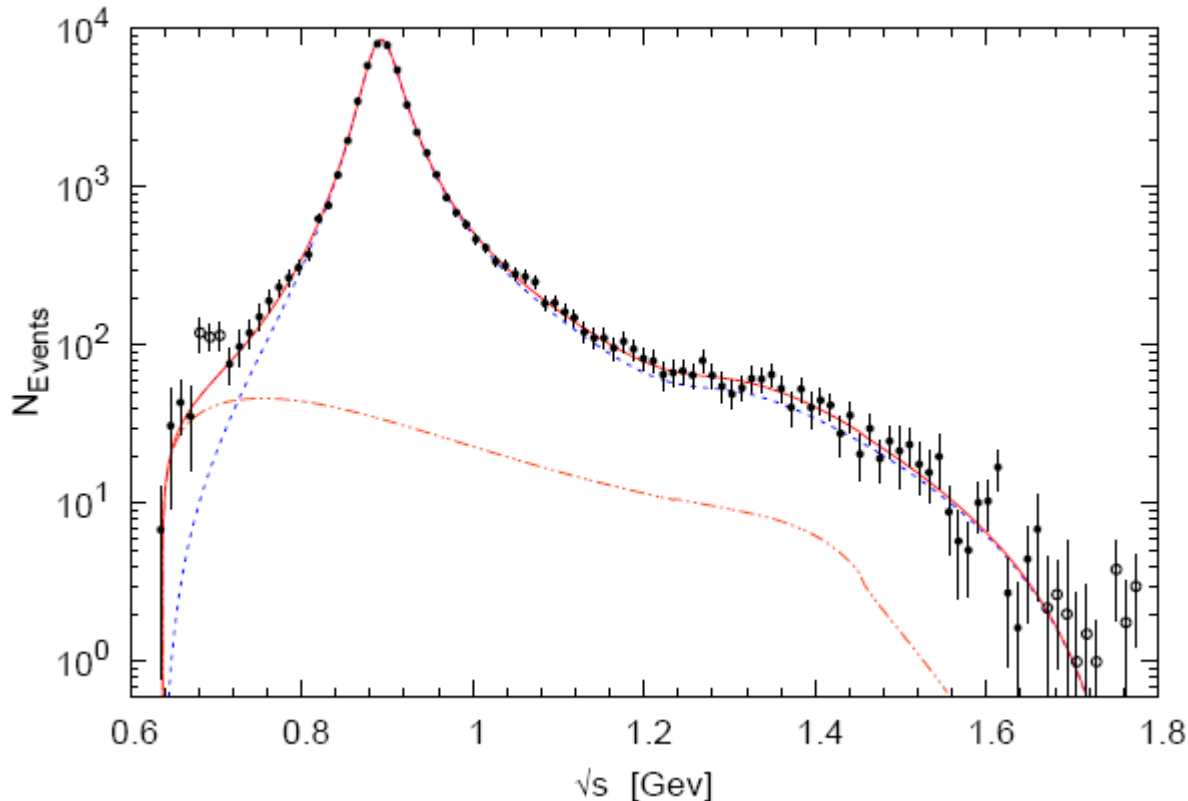
$$\bar{F}_+^{K\pi}(s) = \frac{m_{K^*}^2 - \kappa_{K^*} \bar{A}_{K\pi}(0) + \gamma s}{D(m_{K^*}, \gamma_{K^*})} - \frac{\gamma s}{D(m_{K^{*'}} , \gamma_{K^{*'}})}$$

(Boito, Escribano, Jamin '10)

$$D(m_n, \gamma_n) = m_n^2 - s - \kappa_n \text{Re} \bar{A}_{K\pi}(s) - i m_n \gamma_n(s),$$

$$\gamma_{K^*}(s) = \gamma_{K^*} \frac{s}{m_{K^*}^2} \frac{\sigma_{K\pi}^3(s)}{\sigma_{K\pi}^3(m_{K^*}^2)}, \quad \kappa_{K^*} = \frac{192\pi F F_K}{\sigma_{K\pi}(m_{K^*}^2)^3} \frac{\gamma_{K^*}}{m_{K^*}},$$

$$F_+^{K\pi}(0) = \frac{m_{K^*}^2}{m_{K^*}^2 - \kappa \bar{A}_{K\pi}(0)}$$



Fit to (Belle '07)

Pablo Roig (IFAE)

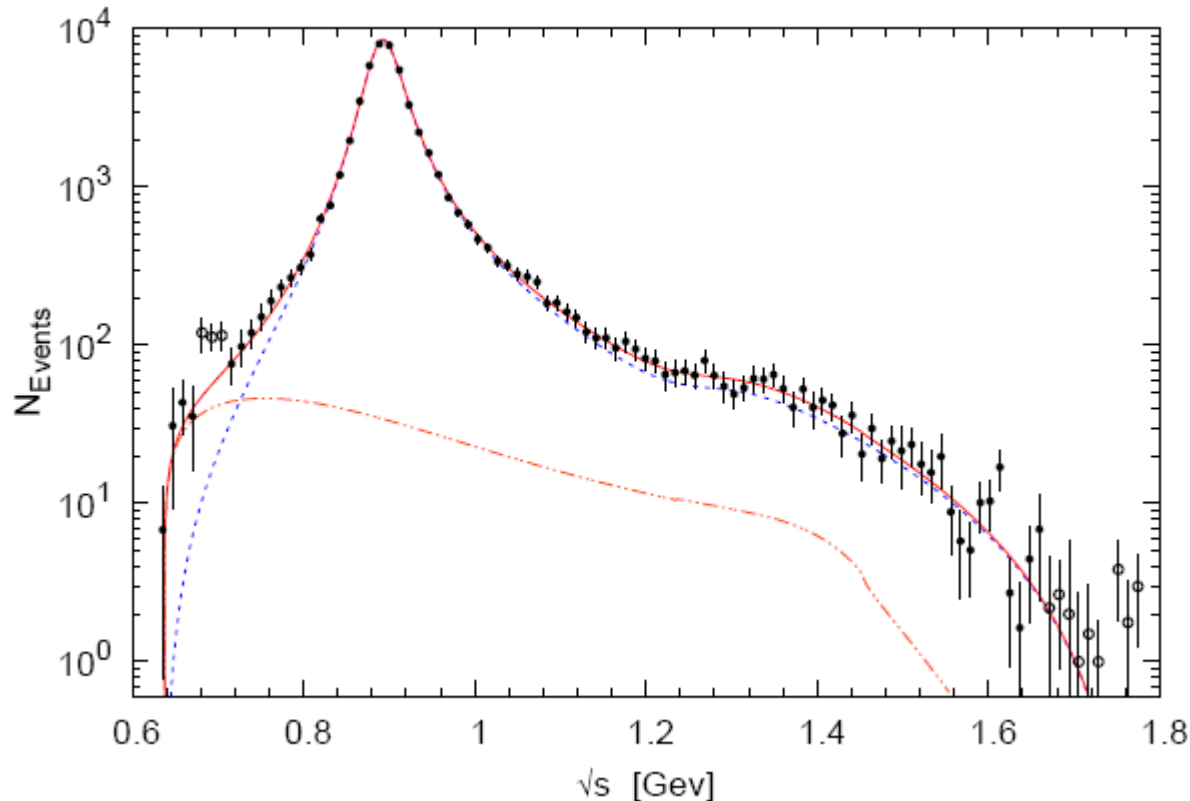
$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in TAUOLA

Other two meson τ decay channels: $(K\pi)^-$, K^-K^0

$$\langle \pi^-(p) | \bar{s} \gamma^\mu u | K^0(k) \rangle = \left[(k+p)^\mu - \frac{m_K^2 - m_\pi^2}{q^2} (k-p)^\mu \right] F_+(q^2) + \frac{m_K^2 - m_\pi^2}{q^2} (k-p)^\mu F_0(q^2)$$

(Jamin, Oller, Pich '02)

F_0 is not implemented yet in the new version of currents in TAUOLA, since there are some problems of stability associated to it.



Fit to (Belle '07)

Pablo Roig (IFAE)

Three meson τ decay channels:

$$(\pi\pi\pi)^-, (KK)\pi^-, K^-K^0\pi^0$$

(Gómez-Dumm, Roig, Pich, Portolés '09)

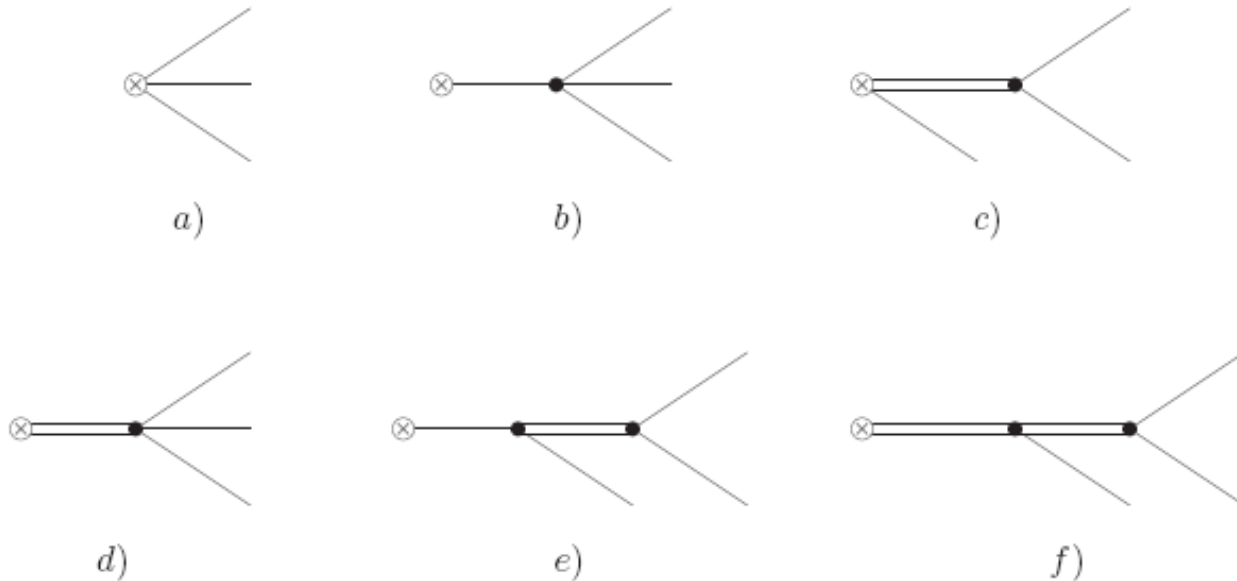


Figure 1: Topologies contributing to the final hadron state in $\tau \rightarrow KK\pi\nu_\tau$ decays in the $N_C \rightarrow \infty$ limit. A crossed circle indicates the QCD vector or axial-vector current insertion. A single line represents a pseudoscalar meson (K, π) while a double line stands for a resonance intermediate state. Topologies $b)$ and $e)$ only contribute to the axial-vector driven form factors, while diagram $d)$ arises only (as explained in the text) from the vector current.

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**

Three meson τ decay channels: $(\pi\pi\pi)^-$

(Gómez-Dumm, Roig, Pich, Portolés '09)

$$F_{\pm i} = \pm (F_i^X + F_i^R + F_i^{RR}) , \quad i = 1, 2 \quad F_2(Q^2, s, t) = F_1(Q^2, t, s)$$

$$F_1^X(Q^2, s, t) = -\frac{2\sqrt{2}}{3F}$$

$$F_1^R(Q^2, s, t) = \frac{\sqrt{2} F_V G_V}{3 F^3} \left[\frac{3s}{s - M_V^2} - \left(\frac{2G_V}{F_V} - 1 \right) \left(\frac{2Q^2 - 2s - u}{s - M_V^2} + \frac{u - s}{t - M_V^2} \right) \right]$$

$$F_1^{RR}(Q^2, s, t) = \frac{4 F_A G_V}{3 F^3} \frac{Q^2}{Q^2 - M_A^2} \left[-(\lambda' + \lambda'') \frac{3s}{s - M_V^2} + H(Q^2, s) \frac{2Q^2 + s - u}{s - M_V^2} + H(Q^2, t) \frac{u - s}{t - M_V^2} \right]$$

$$H(Q^2, x) = -\lambda_0 \frac{m_\pi^2}{Q^2} + \lambda' \frac{x}{Q^2} + \lambda''$$

Relations from short-distance QCD:

$$F_V G_V = F^2$$

$$F_V^2 - F_A^2 = F^2$$

$$F_V^2 M_V^2 = F_A^2 M_A^2$$

$$\lambda' = \frac{F^2}{2\sqrt{2} F_A G_V} = \frac{M_A}{2\sqrt{2} M_V}$$

$$\lambda'' = \frac{2G_V - F_V}{2\sqrt{2} F_A} = \frac{M_A^2 - 2M_V^2}{2\sqrt{2} M_V M_A}$$

$$4\lambda_0 = \lambda' + \lambda'' = \frac{M_A^2 - M_V^2}{\sqrt{2} M_V M_A}$$

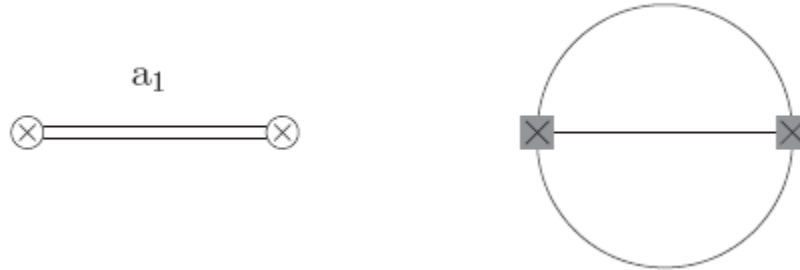
Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in TAUOLA

Three meson τ decay channels: $(\pi\pi\pi)^-$

(Gómez-Dumm, Roig, Pich, Portolés '09)

$\Gamma_{a_1}(Q^2)$:



$$\Gamma_{a_1}(Q^2) = \Gamma_{a_1}^{\pi}(Q^2) \theta(Q^2 - 9m_{\pi}^2) + \Gamma_{a_1}^K(Q^2) \theta(Q^2 - (2m_K + m_{\pi})^2),$$

$$\Gamma_{a_1}^{\pi,K}(Q^2) = \frac{-S}{192(2\pi)^3 F_A^2 M_{a_1}} \left(\frac{M_{a_1}^2}{Q^2} - 1 \right)^2 \int ds dt T_{1+}^{\pi,K\mu} T_{1+\mu}^{\pi,K*}.$$

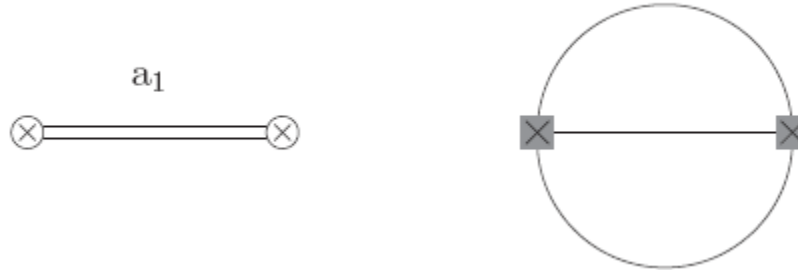
Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_{\tau}$ and other decays of interest in **TAUOLA**

Three meson τ decay channels: $(\pi\pi\pi)^-$

(Gómez-Dumm, Roig, Pich, Portolés '09)

$\Gamma_{a_1}(Q^2)$:



$$\Gamma_{a_1}(Q^2) = \Gamma_{a_1}^{\pi}(Q^2) \theta(Q^2 - 9m_{\pi}^2) + \Gamma_{a_1}^K(Q^2) \theta(Q^2 - (2m_K + m_{\pi})^2),$$

$$\Gamma_{a_1}^{\pi,K}(Q^2) = \frac{-S}{192(2\pi)^3 F_A^2 M_{a_1}} \left(\frac{M_{a_1}^2}{Q^2} - 1 \right)^2 \int ds dt T_{1^+}^{\pi,K\mu} T_{1^+\mu}^{\pi,K*}.$$

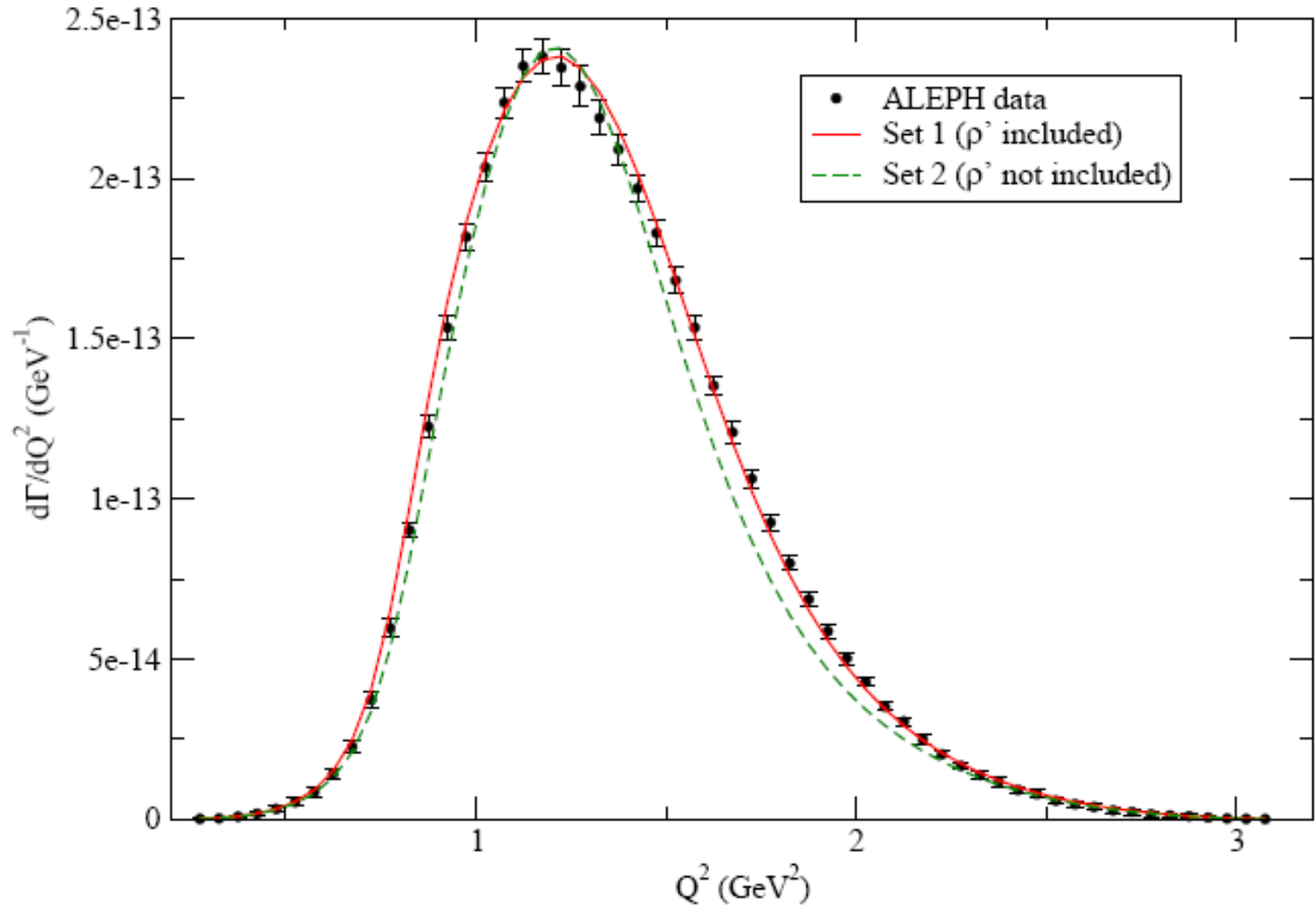
$$\frac{1}{M_{\rho}^2 - q^2 - iM_{\rho}\Gamma_{\rho}(q^2)} \longrightarrow \frac{1}{1 + \beta_{\rho'}} \left[\frac{1}{M_{\rho}^2 - q^2 - iM_{\rho}\Gamma_{\rho}(q^2)} + \frac{\beta_{\rho'}}{M_{\rho'}^2 - q^2 - iM_{\rho'}\Gamma_{\rho'}(q^2)} \right]$$

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_{\tau}$ and other decays of interest in **TAUOLA**

Three meson τ decay channels: $(\pi\pi\pi)^-$

(Gómez-Dumm, Roig, Pich, Portolés '09)



Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in TAUOLA

Three meson τ decays: $(KK)\pi^-$, $K^-K^0\pi^0$

(Gómez-Dumm, Roig, Pich, Portolés '09)

- All four independent form factors have to be taken into account even in the SU(2) case.
- Again, the pseudoscalar form factor, F_4 , is negligible.
- There are many more resonance couplings involved than in the $\pi\pi\pi$ case (all belonging to the odd-intrinsic parity sector).
- We do not have any hint on the value of two of them.
- Up to now, excited resonances have not been implemented.
- Through the framework provided by **TAUOLA**, with the new currents from RcT installed, it will be much easier to learn about the unknown couplings and to estimate properly the size of the excited resonances contribution.

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**

Conclusions and future work

Hadronic currents for the modes: $\pi^-\pi^0$, $(K\pi)^-$, K^-K^0 , $(\pi\pi\pi)^-$, $(KK)\pi^-$, $K^-K^0\pi^0$ - from R χ T have been implemented in TAUOLA (88% of hadronic decays of the tau lepton). They are ready for precise confrontation with data amassed at Belle and BaBar (and future Belle II & Frascati superB data).

In order to obtain the maximum possible information from experiments, the theory input to the MC has to be as accurate as possible with known properties respected (χ PT results at low energies, smooth behaviour of FF at short distances, unitarity, analyticity,...). That is why our effort is and will be worth.

There are improvements to be done in all modes: appropriate SU(2) breaking in $\pi^-\pi^0$, stabilization of scalar form factor in $(K\pi)^-$, inclusion of excited resonances in K^-K^0 , addition of the σ contribution in $(\pi\pi\pi)^-$, and again inclusion of excited resonances in $(KK)\pi^-$ and $K^-K^0\pi^0$.

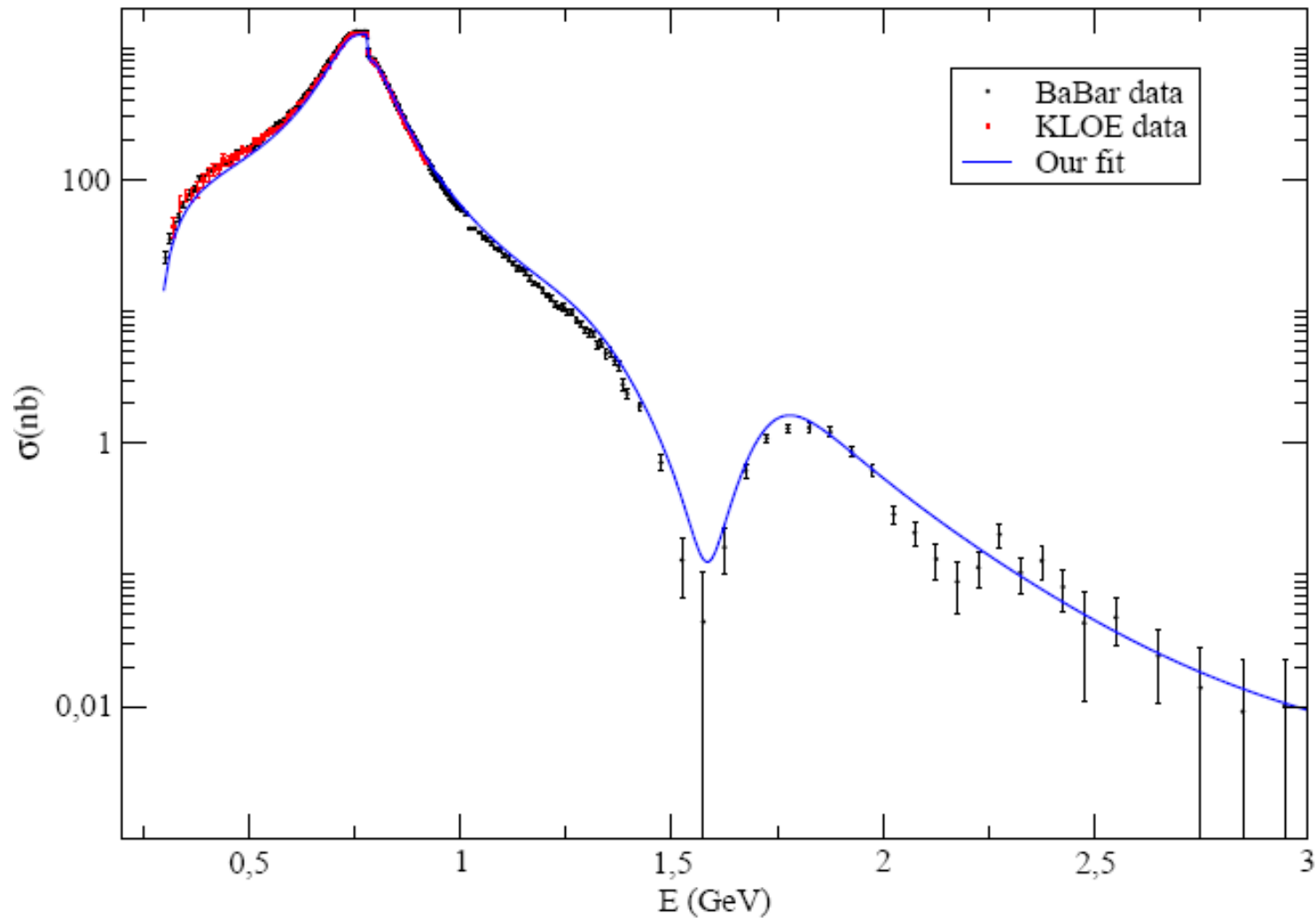
We plan to implement $(K\pi\pi)^-$ and $\eta\pi^-\pi^0$

The study of the relevant mode $(\pi\pi\pi\pi)^-$ will not be tackled immediately.

Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^-\pi^0\nu_\tau$ and other decays of interest in **TAUOLA**

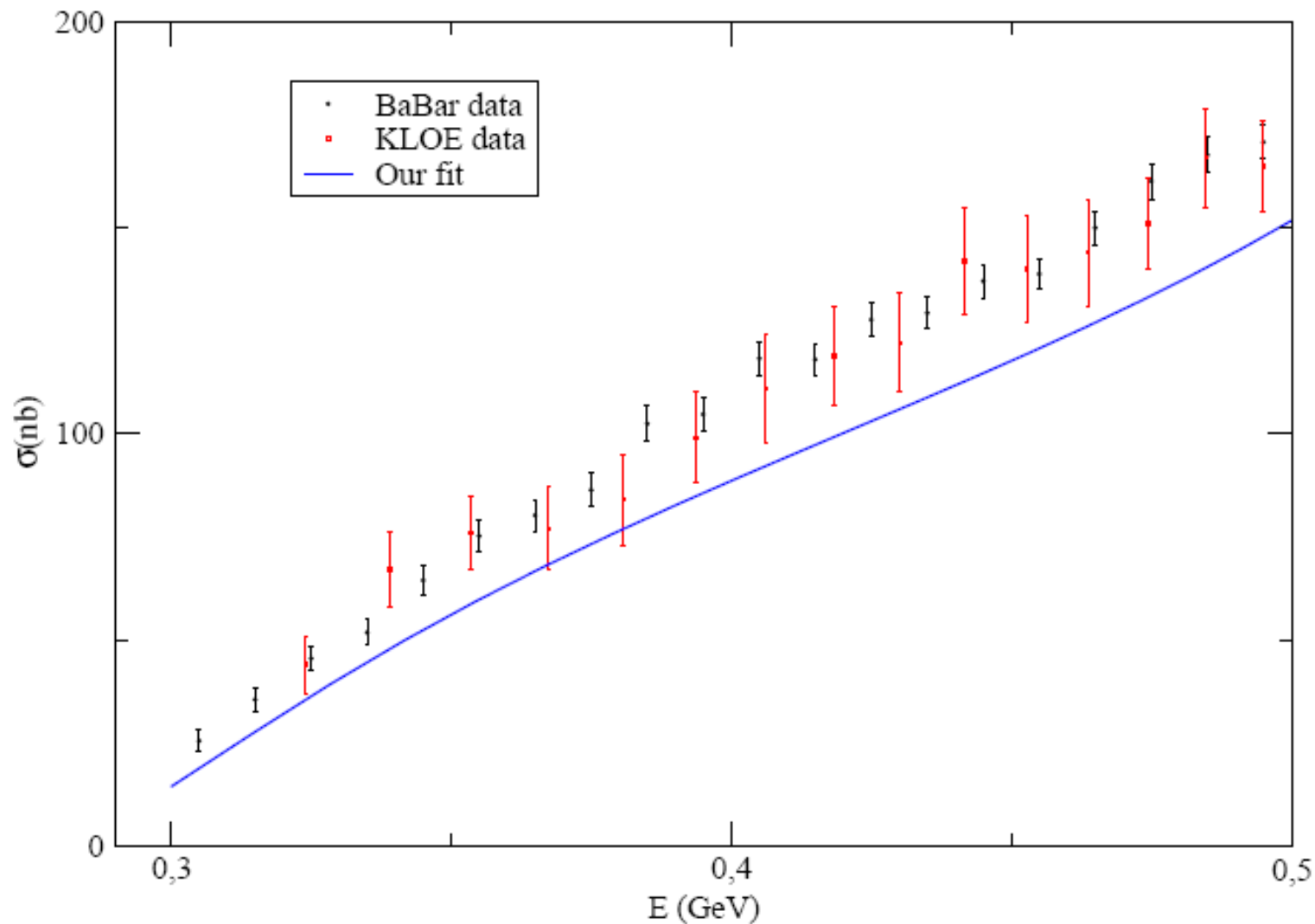
$$e^+e^- \rightarrow \pi^+\pi^-$$



Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**

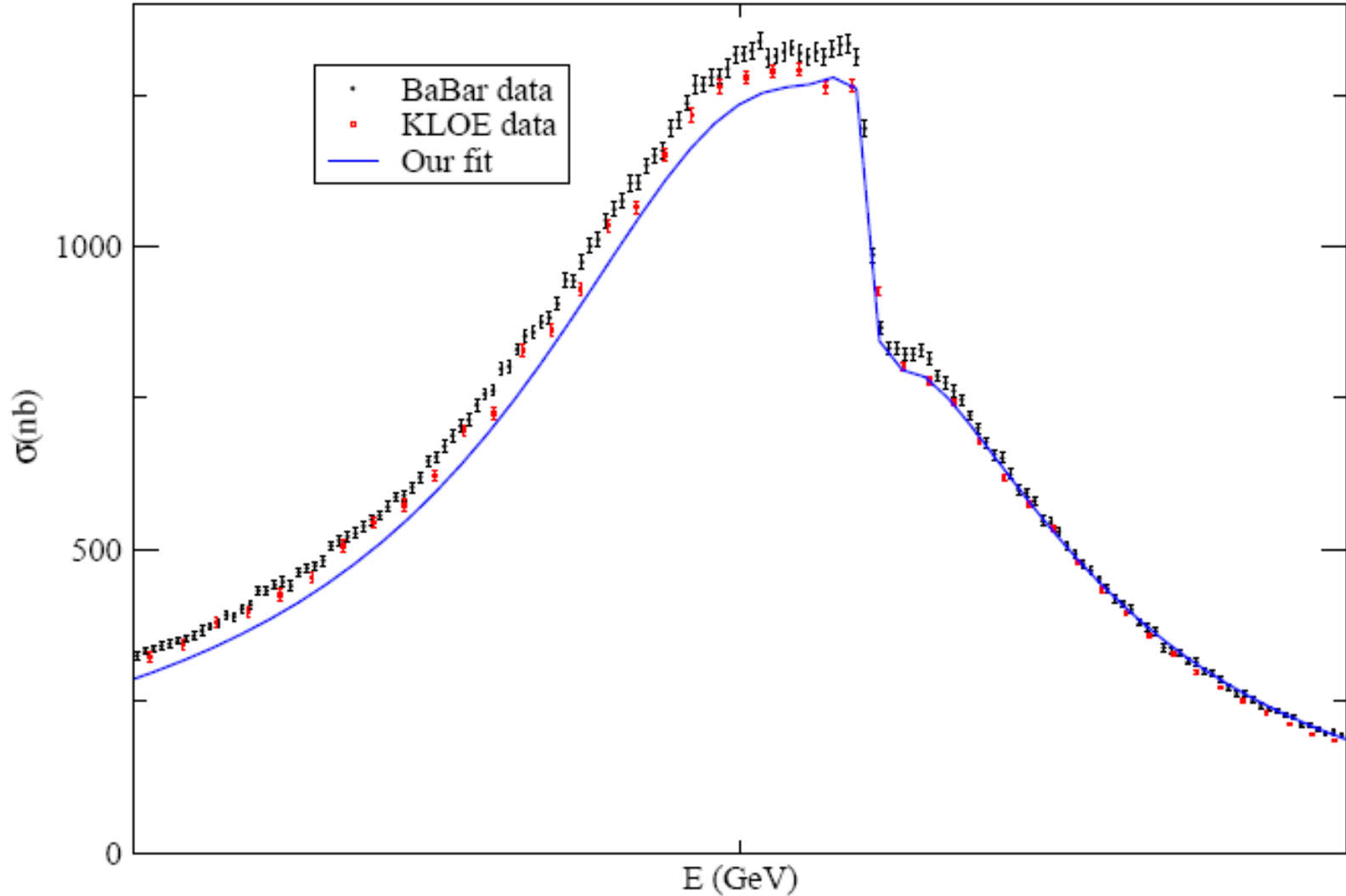
$$e^+e^- \rightarrow \pi^+\pi^-$$



Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**

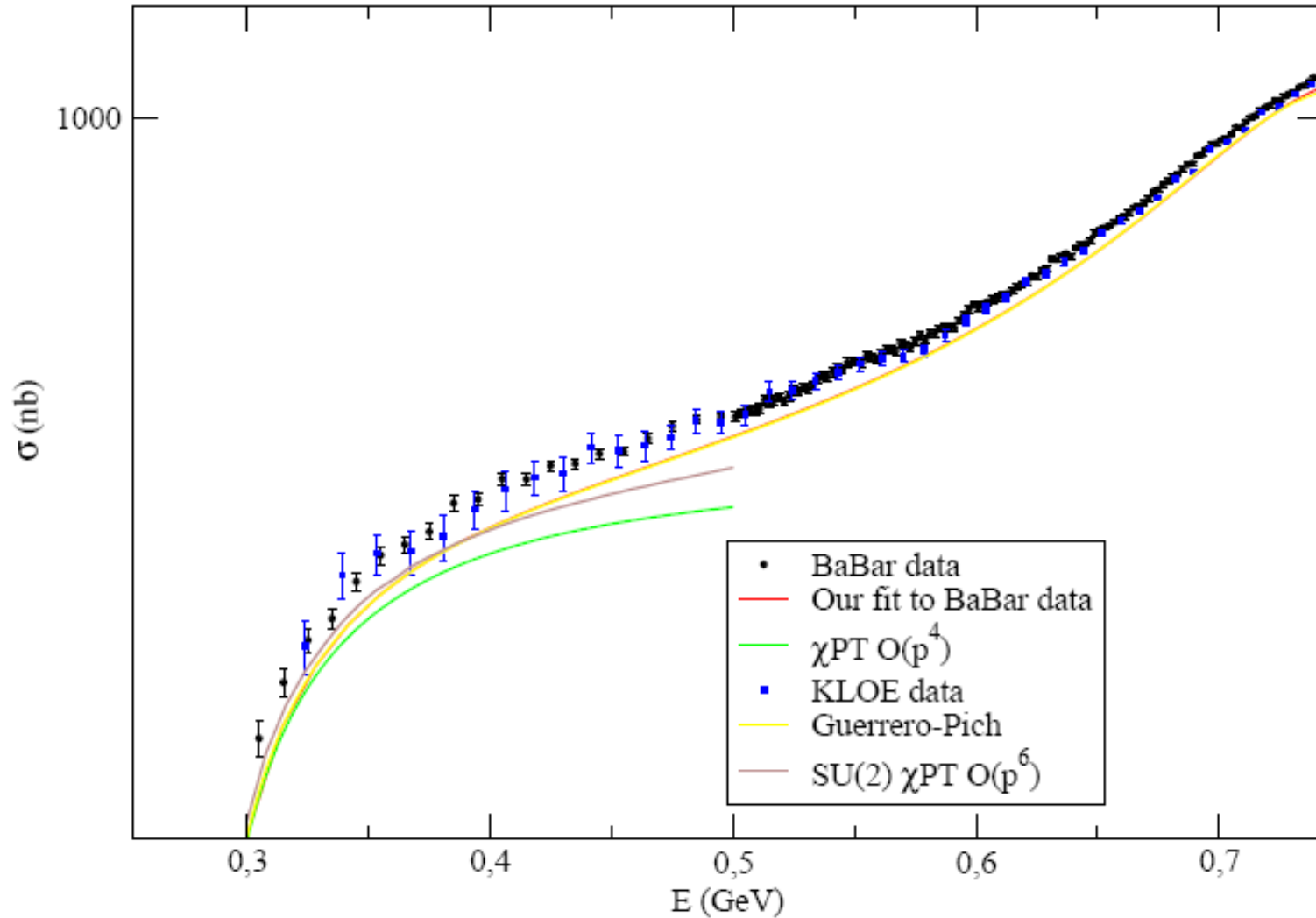
$$e^+e^- \rightarrow \pi^+\pi^-$$



Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**

$$e^+e^- \rightarrow \pi^+\pi^-$$



Pablo Roig (IFAE)

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in **TAUOLA**

Analitycity and unitarity of VFF to $\pi^-\pi^0$

$$\text{Im}F(s) = \text{Im}F(s)_{2\pi} + \text{Im}F(s)_{4\pi} + \dots + \text{Im}F(s)_{K\bar{K}} + \dots$$

(Watson, '52)

$$\text{Im}F(s+i\epsilon) = \sigma_\pi T_1^1 F(s)^* = e^{i\delta_1^1} \sin \delta_1^1 F(s)^* = \sin \delta_1^1 |F(s)| = \tan \delta_1^1 \text{Re}F(s)$$

$$F(s) = \sum_{k=0}^{n-1} \frac{s^k}{k!} \frac{d^k}{ds^k} F(0) + \frac{s^n}{\pi} \int_{4m_\pi^2}^{\infty} \frac{dz}{z^n} \frac{\tan \delta_1^1(z) \text{Re}F(z)}{z-s-i\epsilon}$$

(Omnés, '58)

$$F(s) = Q_n(s) \exp \left\{ \frac{s^n}{\pi} \int_{4m_\pi^2}^{\infty} \frac{dz}{z^n} \frac{\delta_1^1(z)}{z-s-i\epsilon} \right\} \quad \ln Q_n(s) = \sum_{k=0}^{n-1} \frac{s^k}{k!} \frac{d^k}{ds^k} \ln F(0)$$

Matching to $F(s) = \frac{M_\rho^2}{M_\rho^2 - s} - \frac{s}{96\pi^2 f_\pi^2} \left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2} A(m_K^2/s, m_K^2/M_\rho^2) \right]$

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s} \exp \left\{ \frac{-s}{96\pi^2 f_\pi^2} \left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2} A(m_K^2/s, m_K^2/M_\rho^2) \right] \right\}$$

Pablo Roig (IFAE)

$\tau \rightarrow \pi^-\pi^0\nu_\tau$ and other decays of interest in **TAUOLA**