$g - 2$ and New Physics

Daisuke Nomura (Tohoku U.)

talk at Phipsi11@Novosibirsk, September 22, 2011

Based on arXiv:1104.1769 by G.-C. Cho, K. Hagiwara, DN and Yu Matsumoto
**Introduction: Standard Model prediction for muon $g - 2$**

<table>
<thead>
<tr>
<th>Contribution</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>QED contribution</strong></td>
<td>$11 658 471.808 (0.015) \times 10^{-10}$</td>
<td>Kinoshita &amp; Nio, Aoyama et al</td>
</tr>
<tr>
<td><strong>EW contribution</strong></td>
<td>$15.4 (0.2) \times 10^{-10}$</td>
<td>Czarnecki et al</td>
</tr>
<tr>
<td><strong>Hadronic contribution</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LO hadronic</td>
<td>$694.9 (4.3) \times 10^{-10}$</td>
<td>HLMNT11</td>
</tr>
<tr>
<td>NLO hadronic</td>
<td>$-9.8 (0.1) \times 10^{-10}$</td>
<td>HLMNT11</td>
</tr>
<tr>
<td>light-by-light</td>
<td>$10.5 (2.6) \times 10^{-10}$</td>
<td>Prades, de Rafael &amp; Vainshtein</td>
</tr>
<tr>
<td><strong>Theory TOTAL</strong></td>
<td>$11 659 182.8 (4.9) \times 10^{-10}$</td>
<td></td>
</tr>
<tr>
<td><strong>Experiment</strong></td>
<td>$11 659 208.9 (6.3) \times 10^{-10}$</td>
<td>world avg</td>
</tr>
<tr>
<td>Exp — Theory</td>
<td>$26.1 (8.0) \times 10^{-10}$</td>
<td>$3.3 \sigma$ discrepancy</td>
</tr>
</tbody>
</table>

(Numbers taken from HLMNT11, arXiv:1105.3149)

**n.b.: hadronic contributions:**

LO, NLO, and light-by-light Feynman diagrams.

D. Nomura (Tohoku U)
Muons $g-2$:

- Powerful probe for New Physics at TeV scale
- $\sim 3\sigma$ deviation between exp and theory (SM) → Signal of new physics?
- Possible signal from EW sector of New Physics → What about other EW precision tests?
Now, the question is: Suppose that a New Physics is responsible for the muon $g-2$ anomaly. **Is it possible for the New Physics to be compatible with the final LEP EW data?**

— Important question to study BEFORE the LHC

In this talk I take the MSSM as an example of new physics, since it is one of the most attractive models.
Suppose that the $\sim 3\sigma$ deviation is due to SUSY...

Leading SUSY contributions in the $m_Z/m_{\text{SUSY}}$ expansion:

In most cases, the $\tilde{\chi}^{\pm}-\tilde{\nu}$ diagram (a) and/or the $\tilde{B}-\tilde{\mu}_{L/R}$ diagram (b) dominate. (Lopez-Nanopoulos-Wang, Chattopadhyay-Nath, Moroi, ⋯)
MSSM Contributions to Muon $g - 2$

- **x-axis**: $M_2$ (gaugino mass)
- **y-axis**: $m_{\tilde{l}}$ (slepton mass)
The chargino diagram (a) and/or the Bino-smuon \(_{L,R}\) diagram (b) dominate in all the sample points.
### Selected SUSY Models

<table>
<thead>
<tr>
<th></th>
<th>$\tan \beta$</th>
<th>$\mu$</th>
<th>$m_{\tilde{\mu}_L}$</th>
<th>$m_{\tilde{\mu}_R}$</th>
<th>$A_{\mu}$</th>
<th>$M_1$</th>
<th>$M_2$</th>
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<tbody>
<tr>
<td>SG 1 (mSUGRA, $\tan \beta = 10$)</td>
<td>10</td>
<td>396</td>
<td>181</td>
<td>116</td>
<td>$-445$</td>
<td>103</td>
<td>193</td>
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<tr>
<td>SG 2 (mSUGRA, high $\tan \beta$)</td>
<td>50</td>
<td>762</td>
<td>585</td>
<td>465</td>
<td>$-145$</td>
<td>277</td>
<td>510</td>
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<tr>
<td>GM 1 (Gauge Med., high $\tan \beta$)</td>
<td>42</td>
<td>504</td>
<td>441</td>
<td>214</td>
<td>25</td>
<td>181</td>
<td>339</td>
</tr>
<tr>
<td>GM 2 (Gauge Med., $\tan \beta \sim 10$)</td>
<td>15</td>
<td>300</td>
<td>257</td>
<td>120</td>
<td>$-39$</td>
<td>169</td>
<td>327</td>
</tr>
<tr>
<td>MM1 (Mirage Med., $\alpha &gt; 0$)</td>
<td>10</td>
<td>430</td>
<td>188</td>
<td>255</td>
<td>$-465$</td>
<td>170</td>
<td>258</td>
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<tr>
<td>MM2 (Mirage Med., $\alpha &lt; 0$)</td>
<td>10</td>
<td>$-572$</td>
<td>253</td>
<td>108</td>
<td>245</td>
<td>$-99$</td>
<td>$-248$</td>
</tr>
<tr>
<td>MM3 (Mirage Med., $M_2 &lt; M_1$)</td>
<td>10</td>
<td>534</td>
<td>200</td>
<td>237</td>
<td>509</td>
<td>224</td>
<td>173</td>
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### Muon $g - 2$ in the Selected SUSY Models

<table>
<thead>
<tr>
<th></th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(a)-</th>
<th>total</th>
<th>pull</th>
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<tbody>
<tr>
<td>SG 1</td>
<td>25.7</td>
<td>21.5</td>
<td>1.5</td>
<td>-5.2</td>
<td>-5.4</td>
<td>38.1</td>
<td>37.6</td>
<td>1.2</td>
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<tr>
<td>SG 2</td>
<td>20.0</td>
<td>4.8</td>
<td>1.0</td>
<td>-3.4</td>
<td>-2.8</td>
<td>19.5</td>
<td>19.4</td>
<td>-1.0</td>
</tr>
<tr>
<td>GM 1</td>
<td>34.6</td>
<td>11.7</td>
<td>1.4</td>
<td>-5.3</td>
<td>-9.2</td>
<td>33.2</td>
<td>33.0</td>
<td>0.7</td>
</tr>
<tr>
<td>GM 2</td>
<td>27.1</td>
<td>10.6</td>
<td>1.6</td>
<td>-5.0</td>
<td>-9.0</td>
<td>25.3</td>
<td>24.8</td>
<td>-0.3</td>
</tr>
<tr>
<td>MM1</td>
<td>19.4</td>
<td>7.2</td>
<td>1.4</td>
<td>-4.5</td>
<td>-1.9</td>
<td>21.7</td>
<td>21.7</td>
<td>-0.7</td>
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<tr>
<td>MM2</td>
<td>13.2</td>
<td>18.8</td>
<td>0.7</td>
<td>-2.7</td>
<td>-4.2</td>
<td>25.8</td>
<td>24.7</td>
<td>-0.4</td>
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<tr>
<td>MM3</td>
<td>19.6</td>
<td>7.9</td>
<td>1.1</td>
<td>-3.8</td>
<td>-1.8</td>
<td>23.0</td>
<td>23.1</td>
<td>-0.5</td>
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</tbody>
</table>
LEP-I (’89 - ’95): The $Z$-boson properties studied in great detail using 17 millions of $Z$ boson decays. (Final report appeared in 2005: hep-ex/0509008)

To confront the EW precision data with theory, the $S, T, U$ parameters are useful (Peskin+Takeuchi).

\[ \gamma \quad \gamma = i \, \frac{e^2}{\cos^2 \theta_W} \left[ \Pi_{QQ} - s^2 \Pi_{QQ} \right] g^{\mu \nu} + \ldots \]
\[ Z \quad \gamma = i \, \frac{e^2}{c_s^2 s^2} \left[ \Pi_{33} - 2s^2 \Pi_{3Q} + s^4 \Pi_{QQ} \right] g^{\mu \nu} + \ldots \]
\[ Z \quad Z = i \, \frac{e^2}{c_s^2 s^2} \left[ \Pi_{33} - 2s^2 \Pi_{3Q} + s^4 \Pi_{QQ} \right] g^{\mu \nu} + \ldots \]
\[ W \quad W = i \, \frac{e^2}{s^2} \, \Pi_{11} \, g^{\mu \nu} + \ldots \]

\[ \alpha S \equiv 4e^2 \left[ \Pi_{33}^\prime(0) - \Pi_{3Q}^\prime(0) \right], \]
\[ \alpha T \equiv \frac{e^2}{s^2 c^2 m_W^2} \left[ \Pi_{11}(0) - \Pi_{33}(0) \right], \]
\[ \alpha U \equiv 4e^2 \left[ \Pi_{11}^\prime(0) - \Pi_{33}^\prime(0) \right]. \]

In this talk, we use an improved version, $S_Z, T_Z$ and $M_W$ (Hagiwara, Haidt, Kim & Matsumoto).
We define the ‘bar charges’ as

\[
\bar{e}^2(q^2) \equiv \hat{e}^2(\mu) \left[ 1 - \Pi_{T,\gamma}^{\gamma\gamma}(q^2) \right], \quad \bar{s}^2(q^2) \equiv \hat{s}^2(\mu) \left[ 1 + \frac{\hat{c}(\mu)}{\hat{s}(\mu)} \Pi_{T,\gamma}^{\gamma\gamma}(q^2) \right], \\
\bar{g}_Z^2(q^2) \equiv \hat{g}_Z^2(\mu) \left[ 1 - \Pi_{T,Z}^{ZZ}(q^2) \right], \quad \bar{g}_W^2(q^2) \equiv \hat{g}_W^2(\mu) \left[ 1 - \Pi_{T,W}^{WW}(q^2) \right],
\]

where \( \Pi_{T,V}^{AB}(q^2) \equiv \left[ \Pi_{T}^{AB}(q^2) - \Pi_{T}^{AB}(m_V^2) \right]/[q^2 - m_V^2] \) and the hat means the \( \overline{\text{MS}} \) coupling. In terms of the bar charges, the \( S, T \) and \( U \) parameters can be written as

\[
\frac{\bar{s}^2(m_Z^2) \bar{c}^2(m_Z^2)}{\bar{\alpha}(m_Z^2)} - \frac{4\pi}{\bar{g}_Z^2(0)} = \frac{S}{4}, \\
\frac{\bar{s}^2(m_Z^2)}{\bar{\alpha}(m_Z^2)} - \frac{4\pi}{\bar{g}_W^2(0)} = \frac{S + U}{4}, \\
1 - \frac{\bar{g}_W^2(0)}{m_W^2} \frac{m_Z^2}{\bar{g}_Z^2(0)} = \alpha T.
\]
The $S$, $T$ and $U$ parameters:

\[
\frac{\bar{s}^2(m_Z^2)\bar{c}^2(m_Z^2)}{\bar{\alpha}(m_Z^2)} - \frac{4\pi}{\bar{g}_Z^2(0)} = \frac{S}{4},
\]

\[
\frac{\bar{s}^2(m_Z^2)}{\bar{\alpha}(m_Z^2)} - \frac{4\pi}{\bar{g}_W^2(0)} = \frac{S + U}{4},
\]

\[
1 - \frac{\bar{g}_W^2(0)}{m_W^2} \frac{m_Z^2}{\bar{g}_Z^2(0)} = \alpha T.
\]

The last eq. can be written as

\[
\frac{1}{\bar{g}_Z^2(0)} = \frac{1 - \alpha T + \bar{\delta}_G}{4\sqrt{2}G_F m_Z^2},
\]

We are more interested in physics at the $Z$ pole $\Rightarrow$ Replace $\bar{g}_Z(0)$ with $\bar{g}_Z(m_Z^2)$: ($S_Z$ and $T_Z$ parameters)
1. Calculate $O_i^{\text{th}}(\Delta S_Z, \Delta T_Z, \ldots)$, where $O_i$ are EW precision observables ($\Gamma_Z, \sigma^0_h, A_f, \ldots$).

2. Construct the $\chi^2$ function as

$$
\chi^2 = \sum_{i,j} \left( O_i^{\text{th}}(\Delta S_Z, \Delta T_Z, \ldots) - O_i^{\text{exp}} \right) \times \left( V^{-1} \right)_{ij} \left( O_j^{\text{th}}(\Delta S_Z, \Delta T_Z, \ldots) - O_j^{\text{exp}} \right),
$$

where $V$ is the covariance matrix, $V_{ij} = (\delta O_i^{\text{exp}})(\delta O_j^{\text{exp}})\rho_{ij}$.

3. Find the minimum of $\chi^2$ with respect to $\Delta S_Z$, $\Delta T_Z$ etc. Draw the contours $\chi^2 - \chi^2_{\text{min}} = \text{const}$ if necessary.
Using the final LEP EW precision data, we can give a constraint on MSSM contributions to $S_Z$ and $T_Z$.

Our Results:
✓ The SM with $m_H \sim 100$ GeV gives a good description.
✓ In the MSSM, light sfermions tend to be disfavored.
Using the final LEP EW precision data, we can give a constraint on MSSM contributions to $S_Z$ and $T_Z$.

Our Results:
✓ All the sample points are within or close to the 1-$\sigma$ favored region.
In our framework, $M_W$ is a calculable quantity, which can be compared to the data.

**Our Results:**
- Light squarks and sleptons tend to make the fit better.
- Inos do not give contributions to $\Delta m_W$ very much.
- The MSSM with $\mathcal{O}(100)$ GeV SUSY masses gives a good description.

**But this is not the full story...**
The value of the effective mixing angle $\bar{s}^2$ determined from leptonic asymmetry data and that from jet asym. data do not agree very well → problem in jet asym. data (or in the analysis)?
If we do not use the jet asymmetry data, the favored region shifts to the left. (Negative $\Delta S_Z$ is favored.)

✓ Light sleptons are favored.
Constraints from muon $g - 2$ and $b \to s\gamma$
We have studied the favored parameter region of MSSM using the results of the muon $g - 2$ and the EW precision data.

From muon $g - 2$: when $\tan \beta = 10$, the slepton mass of a few hundred GeV is favored. When $\tan \beta = 50$, the sleptons as heavy as 1 TeV are allowed within 1-$\sigma$.

From EW precision data: SUSY particles of a few hundred GeV are OK.

In well-studied models like mSUGRA, Gauge Med. and Mirage Med. there still is some parameter region favored from muon $g - 2$ and EW precision data.

If we leave out the jet asymmetry data, light sleptons become more favored, which is favored from muon $g - 2$ as well.