The light-by-light contribution to the (g-2) of muon from lightest pseudoscalar and scalar mesons within nonlocal chiral quark model

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From ϕ to ψ ,
22 September 2011, BINP, Novosibirsk

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Plan

Motivation

1. Anomalous magnetic momentum of muon $a_\mu = (g-2)_\mu$ is measured in experiment E821(BNL) with high precision

$$a_\mu^{\exp} = 11\,659\,208.9(5.4)(3.3) \cdot 10^{-10}$$

Prediction of Standard Model

$$a_\mu^{\text{theory}} = 11\,659\,183.4(0.2)_{\text{EW}}(4.1)_{\text{Had,LO}}(2.6)_{\text{Had,HO}} \cdot 10^{-10}$$

2. Difference between experiment and prediction of Standard Model is 3 standard deviation

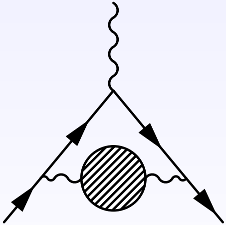
$$a_\mu^{\exp} - a_\mu^{\text{theory}} = 25.5(6.3)(4.9) \cdot 10^{-10}$$

3. The main theoretical error in a_μ^{theory} is due to strong interaction

Motivation. Anomalous magnetic momentum. HVP

4. Contribution of strong interactions can be divided into two parts

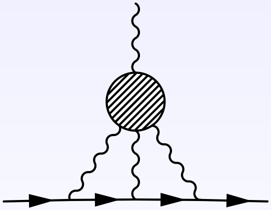
- contribution of hadronic polarization of vacuum (can be extracted from experimental data for process $e^+e^- \to$ in hadrons or hadronic τ-lepton decays)



$$a_\mu^{\mathrm{Had,LO}} = 692.3(4.2) \cdot 10^{-10}$$

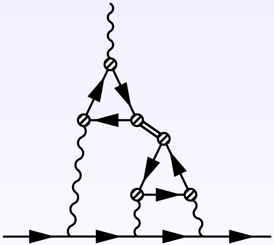
Motivation. Anomalous magnetic momentum. LbL

- light-by-light process



Motivation. Anomalous magnetic momentum. LbL

- light-by-light process



Motivation. LbL. VMD

Different approaches to the calculation of the pseudoscalar meson contributions to the muon AMM from the light-by-light scattering process can be separated in two groups.

1. Various versions of the vector meson dominance model (VMD) that use the hypothesis of the lowest resonances dominance

- **M. Hayakawa and T. Kinoshita**, Phys. Rev. D **57** (1998) 465.
- **M. Knecht and A. Nyffeler**, Phys. Rev. D **65** (2002) 073034.
- **K. Melnikov and A. Vainshtein**, Phys. Rev. D **70** (2004) 113006.
- **A. Nyffeler**, Phys. Rev. D **79** (2009) 073012.
- **L. Cappiello, O. Cata and G. D'Ambrosio**, arXiv:1009.1161 [hep-ph].

Motivation. LbL. EM

2. Effective models that use the dynamical quarks as effective degrees of freedom.

- different versions of the Nambu–Jona-Lasinio model
  J. Bijnens, E. Pallante and J. Prades, Nucl. Phys. B **474**, 379 (1996).
  E. Bartos, A. Z. Dubnickova, S. Dubnicka, E. A. Kuraev and E. Zemlyanaya, Nucl. Phys. B **632** (2002) 330.
- models based on nonperturbative quark-gluon dynamics, like the instanton liquid model
  A. E. Dorokhov and W. Broniowski, Phys. Rev. D **78** (2008) 073011.
- Schwinger-Dyson model
  C. S. Fischer, T. Goecke and R. Williams, Phys. Rev. D **83** (2011) 094006.
- AdS/QCD
  D. K. Hong and D. Kim, Phys. Lett. B **680** (2009)

NχQM Lagrangian, T matrix and $\eta - \eta'$ mixing

The Lagrangian of the nonlocal model has the form

$$\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{4q} + \mathcal{L}_{tH}$$
$$\mathcal{L}_{free} = \bar{q}(x)(i\hat{\partial} - m_c)q(x)$$

m_c – current quark mass matrix with diagonal elements $m_c^u = m_c^d$, m_c^s

$$\begin{aligned}\mathcal{L}_{4q} &= \frac{G}{2}[J_S^a(x)J_S^a(x) + J_P^a(x)J_P^a(x)] \\ \mathcal{L}_{tH} &= -\frac{H}{4}T_{abc}[J_S^a(x)J_S^b(x)J_S^c(x) - 3J_P^a(x)J_P^b(x)J_P^c(x)]\end{aligned}$$

Nonlocal quark currents are

$$J_M^a(x) = \int d^4x_1 d^4x_2\, f(x_1)f(x_2)\,\bar{q}(x - x_1)\,\Gamma_M^a q(x + x_2),$$

where $M = S, P$ and $\Gamma_S^a = \lambda^a$, $\Gamma_P = i\gamma^5\lambda^a$, and $f(x)$ is a form factor reflecting the nonlocal properties of the QCD vacuum.

NχQM **Lagrangian**, T matrix and $\eta - \eta'$ mixing

The model can be bosonized using the stationary phase approximation which leads to the system of gap equations for the dynamical quark masses $m_{d,i} \quad (i = u, d, s)$

$$m_{d,u} + GS_u + \frac{H}{2} S_u S_s = 0,$$
$$m_{d,s} + GS_s + \frac{H}{2} S_u^2 = 0,$$
$$S_i = -8N_c \int \frac{d_E^4 k}{(2\pi)^4} \frac{f^2(k^2) m_i(k^2)}{D_i(k^2)},$$

where $m_i(k^2) = m_{c,i} + m_{d,i} f^2(k^2)$, $D_i(k^2) = k^2 + m_i^2(k^2)$ is the dynamical quark propagator obtained by solving the Schwinger-Dyson equation, $f(k^2)$ is the nonlocal form factor in the momentum representation.

NχQM Lagrangian, $\mathbf{T}$ **matrix** and $\eta - \eta'$ mixing

The vertex functions and the meson masses can be found from the Bethe-Salpeter equation. For the separable interaction the quark-antiquark scattering matrix in pseudoscalar channel becomes

$$\mathbf{T} = \hat{\mathbf{T}}(p^2)\delta^4\left(p_1 + p_2 - (p_3 + p_4)\right)\prod_{i=1}^{4} f(p_i^2),$$

$$\hat{\mathbf{T}}(p^2) = i\gamma_5\lambda_k \left(\frac{1}{-\mathbf{G}^{-1} + \mathbf{\Pi}(p^2)}\right)_{kl} i\gamma_5\lambda_l,$$

where p_i are the momenta of external quark lines, $\mathbf{G}$ and $\mathbf{\Pi}(p^2)$ are the corresponding matrices of the four-quark coupling constants and the polarization operators of pseudoscalar mesons ($p = p_1 + p_2 = p_3 + p_4$). The meson masses can be found from the zeros of determinant $\det(\mathbf{G}^{-1} - \mathbf{\Pi}(-M^2)) = 0$.

NχQM Lagrangian, **T matrix** and $\eta - \eta'$ mixing

The $\hat{\mathbf{T}}$-matrix for the system of pseudoscalar mesons can be expressed in the form

$$\hat{\mathbf{T}}(p^2) = \sum_{a=\pi^0,\eta,\eta'} \frac{\bar{V}_a(p^2) \otimes V_a(p^2)}{-(p^2 + M_a^2)},$$

where M_a are the meson masses, $V_a(p^2)$ are the vertex functions $\left(\bar{V}_a(p^2) = \gamma^0 V_a^\dagger(p^2)\gamma^0\right)$. In general case of three unequal quark masses it is necessary to solve the $\pi^0 - \eta - \eta'$ (or $\eta_0 - \eta_3 - \eta_8$) system factorizing in the isospin limit into the π^0 and $\eta - \eta'$ systems. For the $\eta - \eta'$ system it is convenient to diagonalize the scattering matrix by orthogonal transformation

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix}.$$

NχQM Lagrangian, **T matrix** and $\sigma - f_0$ mixing

The $\hat{\mathbf{T}}$-matrix for the system of scalar mesons can be expressed in the form

$$\hat{\mathbf{T}}(p^2) = \sum_{a=a_0,\sigma,f_0} \frac{\bar{V}_a(p^2) \otimes V_a(p^2)}{-(p^2 + M_a^2)},$$

where M_a are the meson masses, $V_a(p^2)$ are the vertex functions $\left(\bar{V}_a(p^2) = \gamma^0 V_a^\dagger(p^2)\gamma^0\right)$. In general case of three unequal quark masses it is necessary to solve the $a_0 - \sigma - f_0$ (or $\sigma_0 - \sigma_3 - \sigma_8$) system factorizing in the isospin limit into the a_0 and $\sigma - f_0$ systems. For the $\sigma - f_0$ system it is convenient to diagonalize the scattering matrix by orthogonal transformation

$$\begin{pmatrix} \sigma \\ f_0 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \sigma_8 \\ \sigma_0 \end{pmatrix}.$$

NχQM Lagrangian, **T matrix** and $\eta - \eta'$ mixing

The meson mixing angle depends on the virtuality. Therefore $\theta_\eta = \theta(-M_\eta^2)$ and $\theta_{\eta'} = \theta(-M_{\eta'}^2)$ are different for the on-shell η and η' mesons. For numerical estimations we use the Gaussian nonlocal form factor

$$f(k^2) = \exp(-k^2/2\Lambda^2),$$

and the model parameters obtained in [1]. The model parameters (the current quark masses $m_{c,i}$, the coupling constants G and H, and the nonlocality scale Λ) are fixed by requiring that the model reproduces correctly the measured values of the pion and kaon masses, the pion decay constant f_π, and the η' mass (parameter sets G_I, G_{IV}) or the $\eta' \to \gamma\gamma$ decay constant $g_{\eta'\gamma\gamma}$ (sets G_{II}, G_{III}). The sets G_I, G_{IV} vary by different input for the nonstrange current quark mass, while G_{II}, G_{III} are two solutions of the same fitting procedure.

[1] A. Scarpettini, D. Gomez Dumm and N. N. Scoccola, Phys. Rev. D **69** (2004) 114018.

Basic meson properties for different parametrizations

set	M_π [MeV]	M_η [MeV]	$M_{\eta'}$ [MeV]	$g_{\pi\gamma\gamma}$ [GeV^{-1}]	$g_{\eta\gamma\gamma}$ [GeV^{-1}]	$g_{\eta'\gamma\gamma}$ [GeV^{-1}]
G_I	138.9	516.5	958.4	0.2706	0.3082	0.3752
G_{II}	138.9	505.4	878.6	0.2706	0.3259	0.3401
G_{III}	138.9	520.7	1006.4	0.2706	0.3011	0.3489
G_{IV}	139.0	522.1	> 739.7	0.2713	0.3068	
exp	134.9766 ± 0.0006	547.8533 ± 0.024	957.78 ± 0.06	0.2744 $^{+0.009}_{-0.008}$	0.2726 ± 0.008	0.3423 ± 0.014

set	$M_{a_0(980)}$ [MeV]	M_σ [MeV]	$M_{f_0(980)}$ [MeV]	$g_{a_0\gamma\gamma}$ [GeV^{-1}]	$g_{\sigma\gamma\gamma}$ [GeV^{-1}]	$g_{f_0\gamma\gamma}$ [GeV^{-1}]
G_I	796.4	426.6	> 1008	0.032	0.124	
G_{II}	761.4	427.6	> 1008	0.036	0.124	
G_{III}	812.4	426.2	> 1008	0.030	0.124	
G_{IV}	> 739.7	522.1	> 739.7		0.149	
exp	980 ± 20	400–1200	980 ± 10	0.087 $^{+0.017}_{-0.018}$	0.13–1.7	0.086 ± 0.011

Two-photon–pseudoscalar meson. I

Triangular diagram with external pseudoscalar meson and two photon legs with arbitrary virtualities can be written as

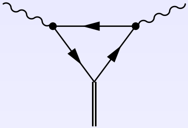
$$A\left(\gamma^*_{(q_1,\epsilon_1)}\gamma^*_{(q_2,\epsilon_2)} \to P^*_{(p)}\right) = -ie^2\varepsilon_{\mu\nu\rho\sigma}\epsilon_1^\mu\epsilon_2^\nu q_1^\rho q_2^\sigma \mathrm{F}_{P^*\gamma^*\gamma^*}\left(p^2;q_1^2,q_2^2\right),$$

$$\mathrm{F}_{\pi_0^*\gamma^*\gamma^*}\left(p^2;q_1^2,q_2^2\right) = g_\pi(p^2)F_u\left(p^2;q_1^2,q_2^2\right),$$

$$\mathrm{F}_{\eta^*\gamma^*\gamma^*}\left(p^2;q_1^2,q_2^2\right) = \frac{g_\eta(p^2)}{3\sqrt{3}}\times$$
$$\times\Big[\left(5F_u\left(p^2;q_1^2,q_2^2\right)-2F_s\left(p^2;q_1^2,q_2^2\right)\right)\cos\theta(p^2)-$$
$$-\sqrt{2}\left(5F_u\left(p^2;q_1^2,q_2^2\right)+F_s\left(p^2;q_1^2,q_2^2\right)\right)\sin\theta(p^2)\Big],$$

$$\mathrm{F}_{\eta'^*\gamma^*\gamma^*}\left(p^2;q_1^2,q_2^2\right) = \frac{g_{\eta'}(p^2)}{3\sqrt{3}}\times$$
$$\times\Big[\left(5F_u\left(p^2;q_1^2,q_2^2\right)-2F_s\left(p^2;q_1^2,q_2^2\right)\right)\sin\theta(p^2)+$$
$$+\sqrt{2}\left(5F_u\left(p^2;q_1^2,q_2^2\right)+F_s\left(p^2;q_1^2,q_2^2\right)\right)\cos\theta(p^2)\Big].$$

Two-photon–pseudoscalar meson. II



$$F_i\left(p^2;q_1^2,q_2^2\right) = 8\int\frac{d_E^4k}{(2\pi)^4}\frac{f(k_1^2)f(k_2^2)}{D_i(k_1^2)D_i(k_2^2)D_i(k^2)}\times$$
$$\times\left[m_i(k^2) - \mathrm{m}_i^{(1)}(k_1,k)J_1 - \mathrm{m}_i^{(1)}(k_2,k)J_2\right],$$
$$J_1 = k^2 + \frac{q_2^2(kq_1)(k_1q_1) - q_1^2(kq_2)(k_1q_2)}{q_1^2q_2^2-(q_1q_2)^2},$$
$$J_2 = k^2 + \frac{q_1^2(kq_2)(k_2q_2) - q_2^2(kq_1)(k_2q_1)}{q_1^2q_2^2-(q_1q_2)^2},$$

where $k_1 = k + q_1$, $k_2 = k - q_2$, $\mathrm{m}_i^{(1)}(k,p) = \frac{m_i(k^2)-m_i(p^2)}{k^2-p^2}$ is the first order finite-difference derivative.

Two-photon–ps meson. III. Special kinematics

1. The kinematics needed for the hadronic exchange LbL

$$F_i\left(q_1^2;q_1^2,0\right) = 8\int\frac{d_E^4k}{(2\pi)^4}\frac{f(k_1^2)f(k^2)}{D_i(k_1^2)D_i^2(k^2)}\times$$
$$\times\left[m_i(k^2) - \mathrm{m}_i^{(1)}(k_1,k)\overline{J}_1 - m_i'(k^2)\overline{J}_2\right],$$
$$\overline{J}_1\left(k,q_1\right) = (kq_1) + \frac{2}{3}\left[k^2 + 2\frac{(kq_1)^2}{q_1^2}\right],$$
$$\overline{J}_2 = \frac{4}{3}\left[k^2 - \frac{(kq_1)^2}{q_1^2}\right],$$

2. Zero momentum kinematics

$$F_i\left(0;0,0\right) = \frac{1}{m_{d,i}}\left[\frac{1}{4\pi^2} - 8m_{c,i}\int\frac{d_E^4k}{(2\pi)^4}\frac{m_i(k^2) - 2m_i'(k^2)k^2}{D_i^3(k^2)}\right],$$

The first term is due to the axial anomaly, while the second term represents the correction due to explicit breaking of the chiral symmetry by current quark mass.

Two-photon–scalar meson. I

Triangular diagram with external scalar meson and two photon legs with arbitrary virtualities can be written as

$$A\left(\gamma^*_{(q_1,\mu)}\gamma^*_{(q_2,\nu)} \to S^*_{(p)}\right) = e^2 \Delta^{\mu\nu}_{S^*\gamma^*\gamma^*}(q_3, q_1, q_2) =$$

$$= e^2 \Big[A_{S^*\gamma^*\gamma^*}\left(p^2; q_1^2, q_2^2\right) T_A^{\mu\nu}(q_1, q_2)$$

$$+ B_{S^*\gamma^*\gamma^*}(p^2; q_1^2, q_2^2) T_B^{\mu\nu}(q_1, q_2) \Big],$$

$$T_A^{\mu\nu}(q_1, q_2) = \left(g^{\mu\nu}(q_1 \cdot q_2) - q_1^\nu q_2^\mu\right)$$

$$T_B^{\mu\nu}(q_1, q_2) = \left(q_1^2 q_2^\mu - (q_1 \cdot q_2) q_1^\mu\right)\left(q_2^2 q_1^\nu - (q_1 \cdot q_2) q_2^\nu\right),$$

Two-photon–scalar meson. Ia

$$\Delta^{\mu\nu}_{a_0^*\gamma^*\gamma^*}\left(p^2;q_1^2,q_2^2\right)=g_{a_0}(p^2)\delta^{\mu\nu}_u\left(p^2;q_1^2,q_2^2\right),$$

$$\Delta^{\mu\nu}_{\sigma^*\gamma^*\gamma^*}\left(p^2;q_1^2,q_2^2\right)=\frac{g_\sigma(p^2)}{3\sqrt{3}}\times$$

$$\times\Big[\left(5\delta^{\mu\nu}_u\left(p^2;q_1^2,q_2^2\right)-2\delta^{\mu\nu}_s\left(p^2;q_1^2,q_2^2\right)\right)\cos\theta(p^2)-$$

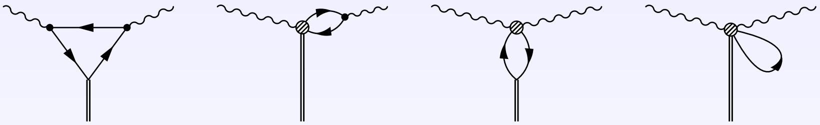
$$-\sqrt{2}\left(5\delta^{\mu\nu}_u\left(p^2;q_1^2,q_2^2\right)+\delta^{\mu\nu}_s\left(p^2;q_1^2,q_2^2\right)\right)\sin\theta(p^2)\Big],$$

$$\Delta^{\mu\nu}_{f_0'^*\gamma^*\gamma^*}\left(p^2;q_1^2,q_2^2\right)=\frac{g_{f_0}(p^2)}{3\sqrt{3}}\times$$

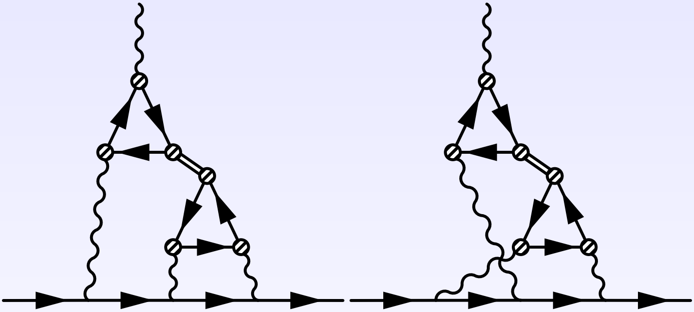
$$\times\Big[\left(5\delta^{\mu\nu}_u\left(p^2;q_1^2,q_2^2\right)-2\delta^{\mu\nu}_s\left(p^2;q_1^2,q_2^2\right)\right)\sin\theta(p^2)+$$

$$+\sqrt{2}\left(5\delta^{\mu\nu}_u\left(p^2;q_1^2,q_2^2\right)+\delta^{\mu\nu}_s\left(p^2;q_1^2,q_2^2\right)\right)\cos\theta(p^2)\Big].$$

Two-photon–scalar meson. II



LbL hadronic contribution to the muon AMM.



LbL hadronic contribution to the muon AMM.

The light-by-light contribution due to exchanges of the light hadrons in the intermediate pseudoscalar channel to the muon AMM can be written in the form

$$a_\mu^{\mathrm{LbL,PS}} = -\frac{2\alpha^3}{3\pi^2}\int_0^\infty dq_1^2 \int_0^\infty dq_2^2 \int_{-1}^{1} dt\sqrt{1-t^2}\frac{1}{q_3^2}\times$$

$$\times \sum_{a=\pi^0,\eta,\eta'}\Bigg[2\frac{\mathrm{F}_{a^*\gamma^*\gamma^*}\left(q_2^2;q_1^2,q_3^2\right)\mathrm{F}_{a^*\gamma^*\gamma}\left(q_2^2;q_2^2,0\right)}{q_2^2+M_a^2}I_1$$

$$+\frac{\mathrm{F}_{a^*\gamma^*\gamma^*}\left(q_3^2;q_1^2,q_2^2\right)\mathrm{F}_{a^*\gamma^*\gamma}\left(q_3^2;q_3^2,0\right)}{q_3^2+M_a^2}I_2\Bigg],$$

where $q_3=-\left(q_1+q_2\right)$. The functions I_1 and I_2 are obtained after averaging over the directions of muon momentum p

$$\langle\ldots\rangle = \frac{1}{2\pi^2}\int d\Omega\left(\widehat{p}\right)\ldots$$

Error bar for π^0 contribution and $SU(2)$ model

It is instructive to investigate the pion contribution to the muon AMM $a_\mu^{\mathrm{LbL},\pi^0}$ for the $SU(2)$ reduction of the nonlocal model.

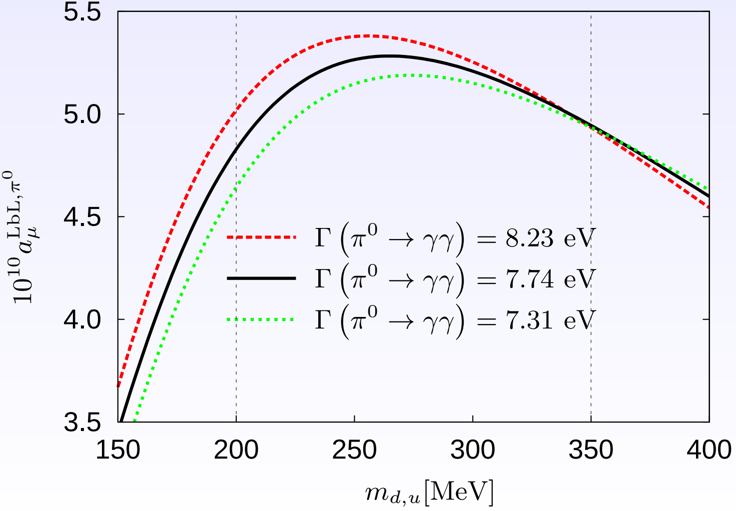
$$\mathcal{L} = \bar{q}(x)(i\hat{\partial} - m_c)q(x) + \frac{G}{2}[J_\sigma(x)J_\sigma(x) + J_\pi(x)J_\pi(x)]$$

Model parameters:

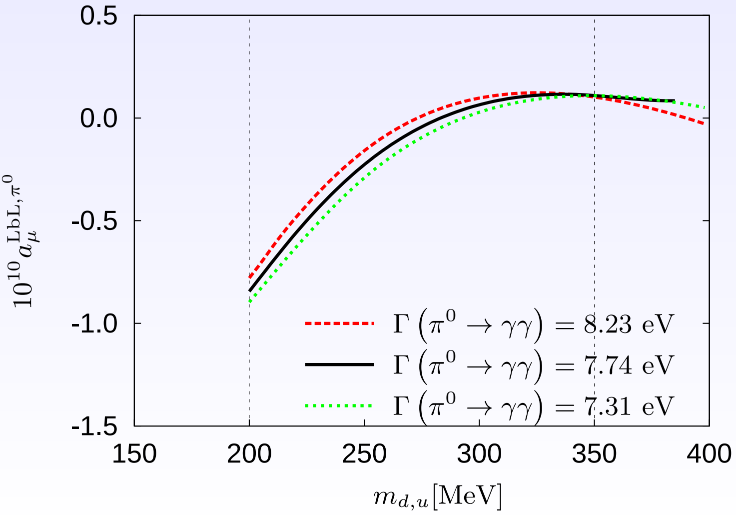
1. current quark mass $m_{c,u}$
2. dynamical quark mass $m_{d,u}$
3. nonlocality parameter Λ

Varying one parameter one can fix other parameters by using as input the pion mass and the two-photon decay constant of the neutral pion.
Dynamical quark mass is taken in the typical interval 200–350 MeV and then other parameters are fitted by the pion mass and the two-photon decay constant in correspondence with the pion lifetime given within the error range of PDG.

LbL contribution to the muon AMM from π^0 and σ exchanges



LbL contribution to the muon AMM from π^0 and σ exchanges



LbL contribution to the muon AMM from π^0 and σ exchanges

Our conservative estimate for π^0 and σ contribution

$$a_\mu^{\mathrm{LbL},\pi^0} = (5.01 \pm 0.37) \cdot 10^{-10}$$
$$a_\mu^{\mathrm{LbL},\sigma} = (-0.9 + 0.12) \cdot 10^{-10}$$

$SU(3)$ model

The contribution of pseudoscalar mesons to the muon AMM $a_\mu^{\rm LbL}$ for different parametrizations. All numbers are given in 10^{-10}.

set	π^0	η	η'	$\eta+\eta'$	$\pi^0+\eta+\eta'$
G_I	5.05	0.55	0.27	0.82	5.87
G_{II}	5.05	0.59	0.48	1.08	6.13
G_{III}	5.05	0.53	0.18	0.71	5.76
G_{IV}	5.10	0.49	0.25	0.74	5.84

For the central value of η and η' contribution we use average over different parametrizations. The error bar for η' is taken as a maximal deviation from central value. The deviation of η contribution from central value seems accidently small, so we use the factor 60% from η' as an estimation of error bar of η contribution.

$SU(3)$ model

Our estimate for η and η' contributions

$$a_\mu^{\mathrm{LbL},\eta} = (0.54 \pm 0.32) \cdot 10^{-10}$$
$$a_\mu^{\mathrm{LbL},\eta'} = (0.30 \pm 0.18) \cdot 10^{-10}$$

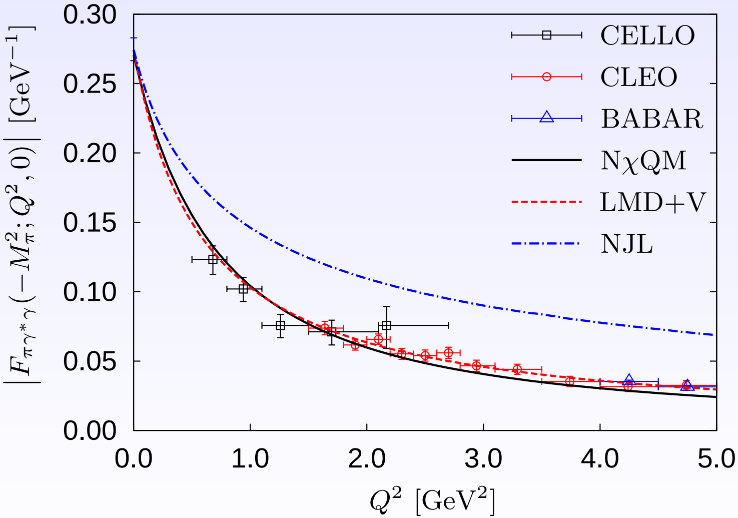
Phenomenological constraints on $P\gamma\gamma$ vertex

1. First phenomenological constraint on the anomalous vertex $\mathrm{F}_{P^*\gamma^*\gamma^*}\left(q_3^2; q_1^2, q_2^2\right)$ is put when both photons and the meson are on-shell. The vertex $\mathrm{F}_{P\gamma\gamma}\left(-M_P^2; 0, 0\right)$ is normalized by the experimentally measured two-photon decay widths

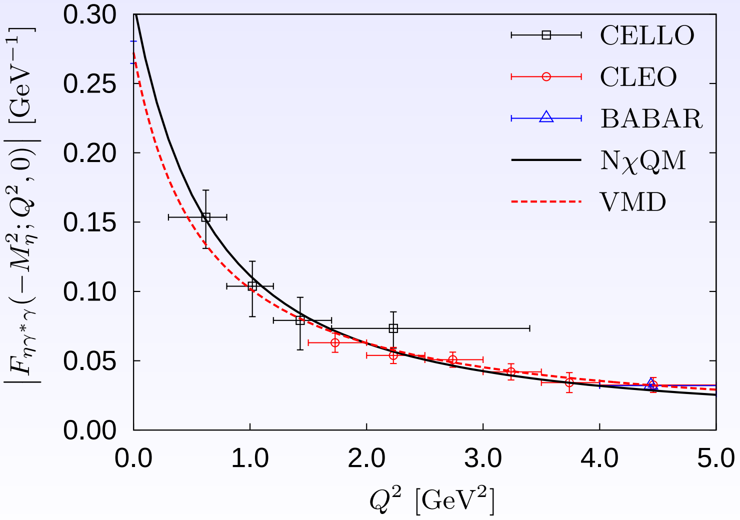
$$\mathrm{F}_{P\gamma\gamma}\left(-M_P^2; 0, 0\right) \equiv g_{P\gamma\gamma} = \sqrt{\frac{64\pi\Gamma\left(P \to \gamma\gamma\right)}{\left(4\pi\alpha\right)^2 M_P^3}}.$$

2. Second phenomenological constraint is, that the vertex for special kinematics, when the meson and one of the photon are on-shell, $\mathrm{F}_{P\gamma\gamma^*}\left(-M_P^2; 0, q^2\right)$, has to fit the data on the pseudoscalar meson transition form factors available from the measurements of the CELLO, CLEO and BABAR collaborations.

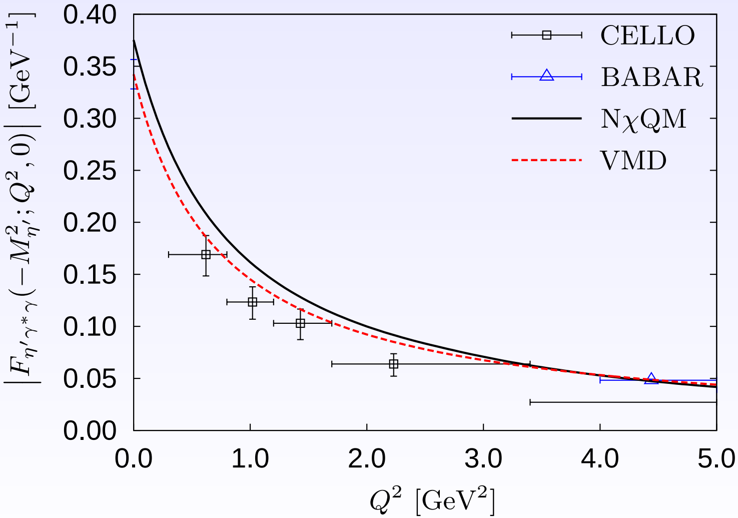
$\gamma^*\gamma \to \pi^0$ transition form factor



$\gamma^*\gamma \to \eta$ transition form factor



$\gamma^*\gamma \to \eta'$ transition form factor



Results and comparison with other models. Pseudoscalars.

Model	π^0	η	η'	$\pi^0 + \eta + \eta'$
VMD [1]	5.74	1.34	1.19	8.27(0.64)
ENJL [2, 3, 4]	5.6			8.5(1.3)
LMD+V [5]	5.8(1.0)	1.3(0.1)	1.2(0.1)	8.3(1.2)
NJL [6]	8.18(1.65)	0.56(0.13)	0.80(0.17)	9.55(1.66)
(LMD+V)′[7]	7.97	1.8	1.8	11.6(1.0)
oLMDV [8]	7.2(1.2)	1.45(0.23)	1.25(0.2)	9.9(1.6)
NχQM [9]	6.5			
HM [10]	6.9	2.7	1.1	10.7
DIP [11]	6.54(0.25)			
DSE [12]	5.75(0.69)	1.36(0.30)	0.96(0.21)	8.07(1.20)
This work (NχQM)	5.01(0.37)	0.54(0.32)	0.30(0.18)	5.85(0.87)

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6. E. Bartos, A. Z. Dubnickova, S. Dubnicka, E. A. Kuraev and E. Zemlyanaya, Nucl. Phys. B **632** (2002) 330.
7. K. Melnikov and A. Vainshtein, Phys. Rev. D **70** (2004) 113006.
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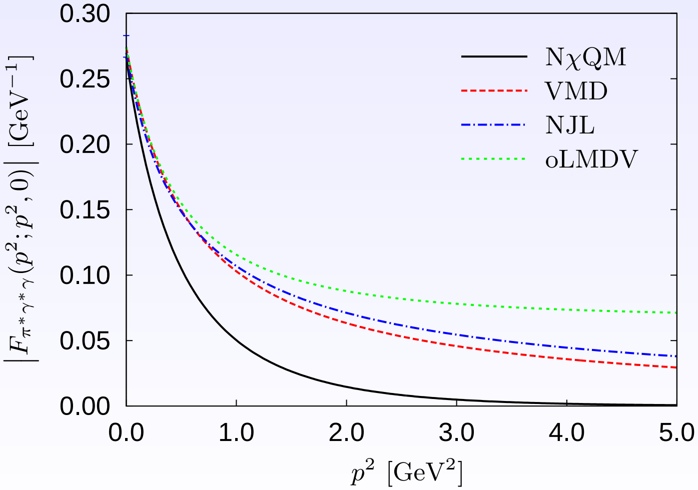
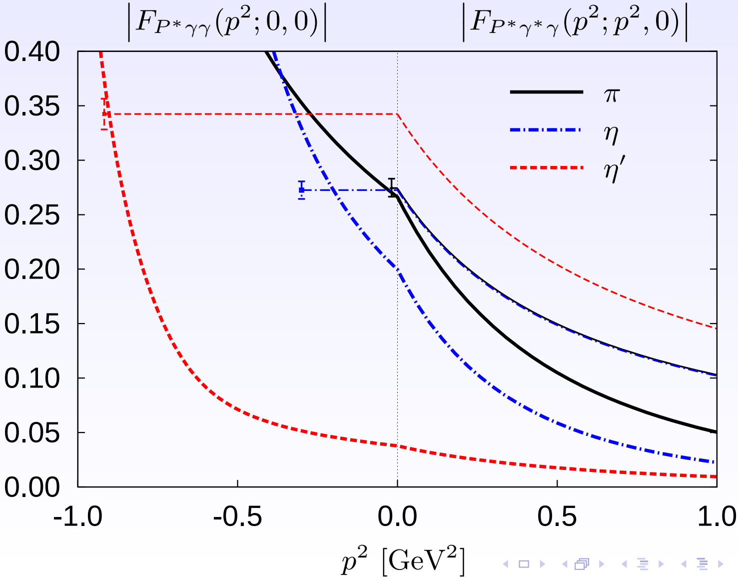
10. D. K. Hong and D. Kim, Phys. Lett. B **680** (2009) 480.
11. L. Cappiello, O. Cata and G. D'Ambrosio, arXiv:1009.1161 [hep-ph].
12. C. S. Fischer, T. Goecke and R. Williams Phys. Rev. D **83** (2011) 094006.

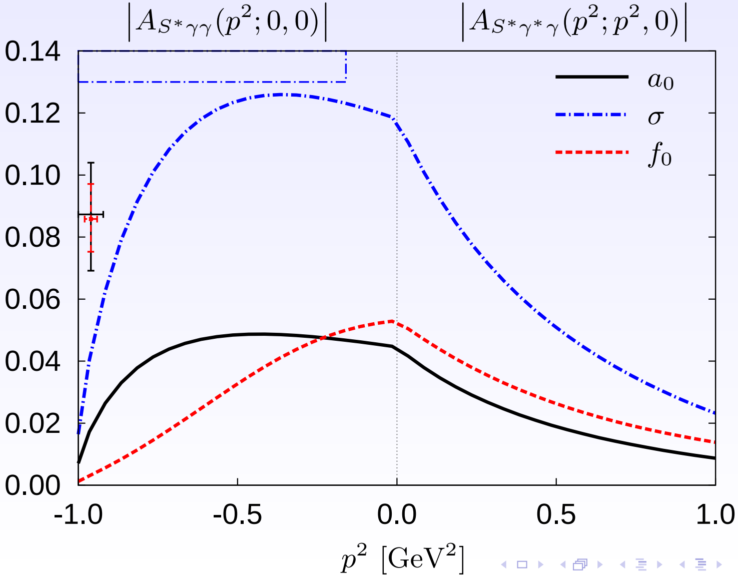
Photon-pion vertex: pion and one photon are off-shell and have the same momenta and another photon is real



$F_{P^*\gamma\gamma}(p^2;0,0)$ in the timelike region and $F_{P^*\gamma^*\gamma}(p^2;p^2,0)$ in the spacelike region



$A_{S^*\gamma\gamma}(p^2;0,0)$ in the timelike region and $A_{S^*\gamma^*\gamma}(p^2;p^2,0)$ in the spacelike region



Conclusions

1. Within the NχQM the pseudoscalar meson contributions to muon AMM are systematically lower then the results obtained in the other works
2. The full kinematic dependence of the vertices on the pion virtuality diminishes the result by about 20-30% as compared to the case where this dependence is neglected
3. For η and η' mesons the results are reduced by factor about 3 in comparison with the results obtained in other models where the kinematic dependence was neglected
4. The total contribution of pseudoscalar exchanges $a_\mu^{\mathrm{PS,LbL}} = (5.85 \pm 0.87) \cdot 10^{-10}$ is approximately factor 1.5 less than the most of previous estimates
5. Scalar contribution can have different sign. It seems that scalar contribution should be partially cancels with box diagram.

THANKS !

LbL hadronic contribution to the muon AMM.

$$I_1 = q_1^2 \Bigg[\left\langle \frac{1}{D_1} \right\rangle \left(\frac{(q_1 q_2)}{2} - q_2^2 \left(1 - t^2\right) \right) + \left\langle \frac{p q_2}{D_1} \right\rangle +$$

$$+ \left\langle \frac{1}{D_1 \cdot D_2} \right\rangle q_2^2 \left(1 - t^2\right) \left(q_2^2 - 2M_\mu^2\right) \Bigg] - \frac{(q_1 q_2)}{2},$$

$$I_2 = 2 q_2^2 \Bigg[\left\langle \frac{1}{D_2} \right\rangle \left(q_1^2 + (q_1 q_2)\right) - \left\langle \frac{p q_1}{D_2} \right\rangle -$$

$$- \left\langle \frac{1}{D_1 \cdot D_2} \right\rangle q_1^2 \left(q_1^2 + (q_1 q_2) + M_\mu^2 \left(1 - t^2\right)\right) \Bigg] +$$

$$+ \left\langle \frac{1}{D_1} \right\rangle q_1^2 (q_1 q_2) - (q_1 q_2),$$

$$D_1 = (p + q_1)^2 + M_\mu^2, \quad D_2 = (p - q_2)^2 + M_\mu^2.$$

LbL hadronic contribution to the muon AMM.

$$\left\langle \frac{1}{D_1} \right\rangle = \frac{R_1 - 1}{2M_\mu^2}, \quad \left\langle \frac{1}{D_2} \right\rangle = \frac{R_2 - 1}{2M_\mu^2},$$

$$\left\langle \frac{1}{D_1 \cdot D_2} \right\rangle = \frac{1}{M_\mu^2 \left|q_1\right| \left|q_2\right| x} \arctan \left[\frac{zx}{1 - zt} \right],$$

$$\left\langle \frac{pq_1}{D_2} \right\rangle = -(q_1 q_2) \frac{(1 - R_2)^2}{8M_\mu^2},$$

$$\left\langle \frac{pq_2}{D_1} \right\rangle = (q_1 q_2) \frac{(1 - R_1)^2}{8M_\mu^2},$$

$$t = \frac{(q_1 q_2)}{\left|q_1\right| \left|q_2\right|}, \quad x = \sqrt{1 - t^2}, \; R_i = \sqrt{1 + \frac{4M_\mu^2}{q_i^2}},$$

$$z = \frac{q_1 q_2}{4M_\mu^2} (1 - R_1)(1 - R_2),$$

and M_μ is the muon mass $\left(p^2 = -M_\mu^2\right)$.

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