Baryon Form Factors at threshold

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PHIPSI11

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Last News on Baryon FF near threshold

The Neutral Baryon Puzzle

Spacelike - Timelike Relationship

Interference Pattern in $J/\psi \rightarrow p\bar{p}$

Conclusions and Perspectives
Cross sections and analyticity

Space-like region
\( eB \to eB \)

FF’s are real

Time-like region
Unphysical region
No data

Data region
\( e^+ e^- \to B\overline{B} \)

FF’s are complex

\[ S_{th} = 4M^2_\pi \]
\[ S_{phy} = 4M^2_B \]

Re\([q^2]\)
Im\([q^2]\)

Time-like: had. helicity
\[
1 \Rightarrow |G_E| \\
0 \Rightarrow |G_M|
\]

\[ G_E(4M^2_B) = G_M(4M^2_B) \]

Elastic scattering
\[
\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E_e' \cos^2 \frac{\theta}{2}}{4E^3_e \sin^4 \frac{\theta}{2}} \left[ G_E^2 - \tau \left( 1 + 2(1 - \tau) \tan^2 \frac{\theta}{2} \right) G_M^2 \right] \frac{1}{1 - \tau}
\]

\[ \tau = \frac{q^2}{4M^2_B} \]

Annihilation
Coulomb correction
\[
\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta C}{4q^2} \left[ (1 + \cos^2 \theta) |G_M|^2 + \frac{1}{\tau} \sin^2 \theta |G_E|^2 \right]
\]

\[ \beta = \sqrt{1 - \frac{1}{\tau}} \]
The Coulomb Factor

\[ \gamma^* \]

\[ B \]

\[ \overline{B} \]

\textbf{Distorted wave approximation}

\[ C = |\psi_{\text{Coul}}(0)|^2 \]

\textbf{S-wave:}

\[ C = \frac{\pi \alpha}{\beta} \frac{1}{1 - \exp\left(-\frac{\pi \alpha}{\beta}\right)} \xrightarrow{\beta \to 0} \frac{\pi \alpha}{\beta} \]

\textbf{D-wave:} \quad C = 1

\textbf{No Coulomb factor for boson pairs (P-wave)}

\[ \sqrt{q^2} \text{ (GeV/c)} \]

\textbf{pp Coulomb interaction as FSI}

[Sommerfeld, Sakharov, Schwinger, Fadin, Khoze]

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Baryon Form Factors at threshold
Sommerfeld Enhancement and Resummation Factors

Coulomb Factor $c$ for S-wave only:

- Partial wave FF: $G_S = \frac{2G_M\sqrt{q^2/4M^2} + G_E}{3}$, $G_D = \frac{G_M\sqrt{q^2/4M^2} - G_E}{3}$

- Cross section: $\sigma(q^2) = 2\pi\alpha^2\beta \frac{4M^2}{(q^2)^2} [c |G_S(q^2)|^2 + 2|G_D(q^2)|^2]$

$C = \mathcal{E} \times \mathcal{R}$

- Enhancement factor: $\mathcal{E} = \pi\alpha/\beta$

- Step at threshold: $\sigma(4M^2) = \frac{\pi^2\alpha^3}{2M^2} \beta^3 |G_S(4M^2)|^2 = 0.85 |G_S(4M^2)|^2$ nb

- Resummation factor: $\mathcal{R} = 1/[1 - \exp(-\pi\alpha/\beta)]$

- Few MeV above threshold: $C \simeq 1 \Rightarrow \sigma(q^2) \propto \beta |G_S(q^2)|^2$
The $e^+e^- \rightarrow \tau^+\tau^-$ case

$\sigma_{\tau\tau}$ (nb) vs $W_{\tau\tau}$ (GeV)

- KEDR
- BES

- With Coulomb corr.
- Without Coulomb corr.
- With only enhancement factor

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Baryon Form Factors at threshold
Pointlike Baryons?

R. Baldini Ferroli, S. Pacetti, A. Zallo and A. Zichichi
Advantages

- All $q$ at the same time $\implies$ Better control on systematics
- c.m. boost $\implies$ at threshold efficiency $\neq 0 + \sigma_W \sim 1 \text{ MeV}$
- Detected ISR $\gamma \implies$ full $pp$ angular coverage

Drawbacks

- $\mathcal{L} \propto$ invariant mass bin $\Delta w$
- More background
Incredibly good at threshold ($\sim 1$ MeV/$c^2$), as $e^+e^-$ c.m.

$\Delta \rho_T/\rho_T \sim 0.5\%$ at 1 GeV
Baryon Form Factors at threshold
Proton form factor at $q^2 = 4M_p^2$

$$\sigma(e^+e^- \rightarrow p\bar{p})(4M_p^2) = 0.83 \pm 0.05 \text{ nb}$$

$$\sigma(e^+e^- \rightarrow p\bar{p})(4M_p^2) = \frac{\pi^2\alpha^3}{2M_p^2} |G^p(4M_p^2)|^2 = 0.85 |G^p(4M_p^2)|^2 \text{ nb}$$

$$|G^p(4M_p^2)| \equiv 1$$

$$|G^p(4M_p^2)| = 0.99 \pm 0.04\text{(stat)} \pm 0.03\text{(syst)}$$
Proton form factor at $q^2 = 4M_p^2$ is

$$|G^p(4M_p^2)| \equiv 1$$

At $q^2 = 4M_p^2$ protons behave as pointlike fermions!
Sommerfeld Resummation Factor Needed?
Resummation Factor Needed?

- At threshold: \( \frac{G_E}{G_M} = 1 \Rightarrow \begin{cases} G_S \in \mathbb{R} \\ G_D = 0 \in \mathbb{R} \end{cases} \)
- \( \sigma(q^2), |\frac{G_E}{G_M}| \rightarrow G_S, G_D \)
- \( G_S = \sqrt{1 - \exp(-\pi \alpha / \beta)} \)
- No need of Resummation Factor

For a wide energy range (~200 MeV):
- Proton behaves as a pointlike particle
- e.m. dominance, no strong interaction?
- Mild sensitivity to \( B\bar{B} \) invariant mass resolution
Babar: $|G_E^p/G_M^p|$ and $\sigma(e^+e^- \rightarrow p\bar{p})$
Babar: $|G_E^p/G_M^p|$ and $\sigma(e^+e^- \rightarrow p\bar{p})$
Babbar: $G_S$, $G_D$ and $G_{\text{eff}}$

Baryon Form Factors at threshold

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PRD73, 012005
The graph represents the integrated Sommerfeld factor, given by:

\[
\frac{1}{\Delta W} \int_{0}^{\Delta W} [1 - \exp(-\pi \alpha / \beta)] dw
\]

The x-axis represents \(\Delta W\) in MeV, ranging from 0 to 4, and the y-axis represents the integrated factor ranging from 1 to 0.4.
Other charged baryon FF’s at threshold
$e^+ e^- \rightarrow \Lambda_c^+ \Lambda_c^-$ and $e^+ e^- \rightarrow p \bar{N}(1440) + c.c.$

[Belle PRL101, 172001]

[BaBar PRD73, 012005]

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Baryon Form Factors at threshold
$e^+ e^- \rightarrow p \bar{N}(1440) + \text{c.c.}$

\[
\sigma_{\text{Coulomb}} = \frac{16 \pi^2 \alpha^3 M_p^3 M_{N(1440)}^3}{(M_p + M_{N(1440)})^5} |G_{pN(1440)}|^2 = |G_{pN(1440)}|^2 \times 0.49 \text{ nb}
\]

\[
|G_{pN(1440)}| = 1.04 \pm 0.09
\]

![Graph showing events distribution](chart1.png)

![Graph showing coupling](chart2.png)
The neutral baryons puzzle
Neutral Baryons puzzle (BABAR)

\[ \sigma(e^+ e^- \rightarrow B^0 \bar{B}^0) = \frac{4\pi\alpha^2/\beta C_0}{3q^2} \left[ |G_M^{B_0}|^2 + \frac{2M_{B_0}^2}{q^2} |G_E^{B_0}|^2 \right] \rightarrow \frac{\pi\alpha^2/\beta}{2M_{B_0}^2} |G_{B^0}|^2 \rightarrow 0 \]

No Coulomb correction at hadron level: \( C_0 = 1 \)

Like a remnant of Coulomb interactions at quark level?

\[ \mathbf{C}_0 \propto \beta^{-1} \text{ as } \sqrt{q^2} \rightarrow 2M_{B^0} \]

For any neutral baryon

\[ \sqrt{\sigma_{B^0 \bar{B}^0}} \propto \frac{|G_{B^0}|}{M_{B^0}} \]
Baryon octet and $U$-spin

$YI_3$ pn $\Sigma^-$ $\Sigma^0$ $\Sigma^+$ $\Lambda$ $\Xi^-$ $\Xi^0$ $\Xi^+$ $U_3$ $Y_U = -Q$

$U_3 = -\frac{1}{2} I_3 + \frac{3}{4} Y$

$G^{\Sigma^0} - G^\Lambda + \frac{2}{\sqrt{3}} G^{\Lambda \Sigma^0} = 0$

$M_{\Sigma^0} \sqrt{\sigma_{\Sigma^0 \Sigma^0}} - M_{\Lambda} \sqrt{\sigma_{\Lambda \Lambda}} + \frac{2}{\sqrt{3}} M_{\Lambda \Sigma^0} \sqrt{\sigma_{\Lambda \Sigma^0}} = (-0.06 \pm 6.0) \times 10^{-4}$
Baryon octet and $U$-spin

\[(Y, I_3) \rightarrow (Y_U, U_3)\]

\[U_3 = -\frac{1}{2} I_3 + \frac{3}{4} Y\]

\[Y_U = -Q\]

**U-spin relation:**

\[G^{\Sigma^0} - G^\Lambda + \frac{2}{\sqrt{3}} G^\Lambda\Sigma^0 = 0\]

\[M_{\Sigma^0} \sqrt{\sigma_{\Sigma^0\Sigma^0}} - M^\Lambda \sqrt{\sigma_{\Lambda\Lambda}} + \frac{2}{\sqrt{3}} M^\Lambda\Sigma^0 \sqrt{\sigma_{\Lambda\Sigma^0}} = (-0.06 \pm 6.0) \times 10^{-4}\]
$e^+ e^- \rightarrow \Lambda \bar{\Lambda}$

$\sigma_{\Lambda\Lambda}$ (nb) vs. $W_{\Lambda\Lambda}$ (GeV)

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Baryon Form Factors at threshold
No Coulomb correction

\[ |G^n_M(q^2)| \]

- FENICE
- DM2
- DM2 extr. from \( G^\Lambda \)

\[ |G^p_M(q^2) \cdot Q_d/ Q_u| \]

\[ \sqrt{q^2} \text{ (GeV)} \]

| \( |G^n_M / G^p_M| \) |
|---|
| Data \quad \sim 1.5 |
| Naively \quad \sim |Q_d / Q_u| |
| pQCD \quad < 1 |
| Soliton models \quad \sim 1 |
| VMD (Dubnicka) \quad \gg 1 |

Only SND, CMD2(?) and BESIII can measure this cross section.

No other experiments at present and in near future will be able to perform such a measurement.
$e^+e^- \rightarrow n\bar{n}$: preliminary result from SND

- **Scan 2011**
- **Maximum energy**: 2 GeV
- **Efficiency**: $\sim 30\%$
- **Above $n\bar{n}$ threshold**: $\sigma_{n\bar{n}} = 0.8 \pm 0.2$ nb

$\sigma_{n\bar{n}}$ (nb) vs. $\sqrt{q^2}$ (GeV)

SND preliminary result for $e^+e^- \rightarrow n\bar{n}$ compared to FENICE (1988).
Dispersive analysis of the ratio $R = \mu_p \frac{G_E^p}{G_M^p}$

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Re($q^2$)

space-like

unphysical region

time-like
Space-like $G_E^p / G_M^p$ measurements

$$G_E^p = F_1^p + \frac{q^2}{4M_p^2} F_2^p$$
$$G_M^p = F_1^p + F_2^p$$

$\mu^p G_E^p(q^2) / G_M^p(q^2)$

$-q^2$ (GeV$^2$/c$^2$)

PRD50 5491

Space-like

$F_1 / \frac{q^2}{4M_p^2} F_2$ cancellation

$$\frac{G_E^p(q^2)}{G_M^p(q^2)} < 1$$

Time-like

$F_1 / \frac{q^2}{4M_p^2} F_2$ enhancement

$$\left| \frac{G_E^p(q^2)}{G_M^p(q^2)} \right| > 1$$

Radiative corrections of polarization technique

Radiative corrections in Rosenbluth method

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Baryon Form Factors at threshold
Space-like $G_E^p / G_M^p$ measurements

\[ G_E^p = F_1^p + \frac{q^2}{4M_p^2} F_2^p \]

\[ G_M^p = F_1^p + F_2^p \]

**Radiative corrections of polarization technique**

**Radiative corrections in Rosenbluth method**

**Space-like**

\[ \frac{G_E^p(q^2)}{G_M^p(q^2)} < 1 \]

**Time-like**

\[ \left| \frac{G_E^p(q^2)}{G_M^p(q^2)} \right| > 1 \]
Time-like $|G_E^p / G_M^p|$ measurements

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2\beta C}{2q^2} |G_M^p|^2 \left[ (1 + \cos^2\theta) + \frac{4M_p^2}{q^2\mu_p^2} \sin^2\theta |R|^2 \right]$$

$$R(q^2) = \mu_p \frac{G_E^p(q^2)}{G_M^p(q^2)}$$

Scaling

**Babbar** (ISR)
PRD73, 012005

**LEAR** ($p\bar{p} \rightarrow e^+ e^-$)
NPB411, 3

**Fenice+DM2**

**E835**
EPJC46, 421

$\gamma\gamma$ exchange

$\gamma\gamma$ exchange interferes with the Born term

Asymmetry in angular distributions

[PLB659, 197]
\[ \mathcal{A}(\cos \theta, q^2) = \frac{d\sigma}{d\Omega}(\cos \theta, q^2) - \frac{d\sigma}{d\Omega}(-\cos \theta, q^2) \]

\[ = \frac{d\sigma}{d\Omega}(\cos \theta, q^2) + \frac{d\sigma}{d\Omega}(-\cos \theta, q^2) \]

\[ \langle \mathcal{A} \rangle_{\cos \theta, q^2} = 0.01 \pm 0.02 \]
$R(q^2)$ in the complex plane

$G_E$, $G_M$ and also $R$, if $G_M$ has no zeros, are analytic on the $q^2$ plane with a cut ($s_{th} = 4M_{\pi}^2$, $\infty$)

[see e.g.: Eur. Phys. J. C 11, 709 (1999)]

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Baryon Form Factors at threshold
Dispersion relation for the imaginary part \((q^2 \leq s_{th})\)

\[
G(q^2) = \lim_{R \to \infty} \frac{1}{2\pi i} \oint_C \frac{G(z)dz}{z - q^2} = \frac{1}{\pi} \int_{s_{th}}^{\infty} \frac{\text{Im} G(s) ds}{s - q^2}
\]
$R(q^2)$ in the complex plane

Dispersion relation for $R$ with subtraction at $q^2 = 0$

$$ R(q^2) = R(0) + \frac{q^2}{\pi} \int_{s_{th}}^{\infty} \frac{\text{Im} R(s) \, ds}{s(s - q^2)} $$

path $C$

|Re($q^2$)|

|Im($q^2$)|

physical sheet

unphysical sheet

experimental sheet
\[ R(q^2) = R(0) + \frac{q^2}{\pi} \int_{4M_{\pi}^2}^{\infty} \frac{\text{Im}R(s)}{s(s - q^2)} \, ds \]

- **R(q^2) space-like**
- **|R(q^2)| time-like**

**JLab+MIT-Bates**

**BABAR+DM2/FENICE+E835**

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Baryon Form Factors at threshold
\[ R(q^2) = R(0) + \frac{q^2}{\pi} \int_{4M^2_\pi}^{\infty} \frac{\text{Im}R(s)}{s(s - q^2)} ds \]

- \( R(q^2) \) space-like
- \( |R(q^2)| \) time-like

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Baryon Form Factors at threshold
\[ R(q^2) = R(0) + \frac{q^2}{\pi} \int_{4M_P^2}^{\infty} \frac{\text{Im}R(s)}{s(s-q^2)} ds \]

**R(q^2) space-like**

**|R(q^2)| time-like**

- **JLab+MIT-Bates**
- **BABAR+DM2/FENICE+E835**

- **DR Approach**
- **1/Q**
- **\( \log^2 Q^2/Q^2 \)**
- **Impr. \( \log^2 Q^2/Q^2 \)**
- **I JL**

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**Baryon Form Factors at threshold**

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\[ R(q^2) = R(0) + \frac{q^2}{\pi} \int_{4M^2_\pi}^{\infty} \frac{\text{Im} R(s)}{s(s - q^2)} ds \]

**$R(q^2)$ space-like**

**$|R(q^2)|$ time-like**

**JLab+MIT-Bates**

**JLab preliminary**

V. Punjabi DSPIN-09
Dubna, Russia

**BABAR+DM2/FENICE+E835**

- DR Approach
- $1/Q$
- $\log^2 Q^2 / Q^2$
- Improt. $\log^2 Q^2/Q^2$
- IJL

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Baryon Form Factors at threshold
Asymptotic $G^p_E(q^2)/G^p_M(q^2)$ and phase

\[ \frac{G^p_E(q^2)}{G^p_M(q^2)} \bigg|_{q^2 \to \infty} \to -1 \]

**pQCD prediction**

**Phase from DR**

\[ \phi(q^2) = -\frac{\sqrt{q^2 - s_0}}{\pi} \text{Pr} \int_{s_0}^{\infty} \frac{\ln|R(s)|}{\sqrt{s - s_0(s - q^2)}} ds \]

Phragmèn Lindelöf phase limit $\leftrightarrow$ zeros

Baryon Form Factors at threshold
$J/\psi$ strong and electromagnetic phase
**BESIII preliminary results: $J/\psi \rightarrow p\bar{p}, n\bar{n}$**

**$n\bar{n}$ identification**

![Graphs showing $E(\bar{n})$ vs. $E(n)$, number of $n$ in 50° cone, and angle between $n$ and recoil dir. of $\bar{n}$](image)

- **BESIII**
  - $B(J/\psi \rightarrow n\bar{n}) = (2.07 \pm 0.01 \pm 0.14) \cdot 10^{-3}$
  - $B(J/\psi \rightarrow p\bar{p}) = (2.112 \pm 0.004 \pm 0.027) \cdot 10^{-3}$

- **PDG**
  - $B(J/\psi \rightarrow n\bar{n}) = (2.2 \pm 0.4) \cdot 10^{-3}$
  - $B(J/\psi \rightarrow p\bar{p}) = (2.17 \pm 0.07) \cdot 10^{-3}$

**$B(J/\psi \rightarrow p\bar{p}) \simeq B(J/\psi \rightarrow n\bar{n})$**
suggests a phase $\sim 90^\circ$ between strong and e.m. amplitudes!
$J/\psi$ decays: strong and electromagnetic

The cross section is given by

$$|A_\gamma + A_{3g}|^2 = |A_\gamma|^2 + |A_{3g}|^2 + 2 \text{Re}[A_\gamma^* A_{3g}]$$

According to pQCD: $A_\gamma$ and $A_{3g}$ are real $\Rightarrow$ interference

On the contrary data suggest:

<table>
<thead>
<tr>
<th>$J/\psi \rightarrow J_1^P J_2^P$</th>
<th>$\frac{A_\gamma}{A_{3g}}$ phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1^- 0^-$</td>
<td>$106^o \pm 10^o$</td>
</tr>
<tr>
<td>$1^- 1^-$</td>
<td>$138^o \pm 37^o$</td>
</tr>
<tr>
<td>$0^- 0^-$</td>
<td>$90^o \pm 10^o$</td>
</tr>
<tr>
<td>$n\bar{n}$</td>
<td>$89^o \pm 15^o$</td>
</tr>
</tbody>
</table>

But these conclusions have been obtained modeling SU(3) breaking, or using poorly measured $n\bar{n}$ cross section outside $J/\psi$

Interference with the continuum measures the relative phase in an independent way
Full interference as seen by PANDA or BESIII

PANDA (\(\Delta p / p = 10^{-4}\))

BESIII

\(\sigma_{pp} (\text{nb})\)

\(W_{pp} (\text{MeV})\)

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Baryon Form Factors at threshold
Conclusions

- Pointlike Behavior at and well above threshold
- No Sommerfeld Resummation Factor
- Neutral baryon non zero cross section at threshold?
- $G_E^{p}$ space-like $\rightarrow -1$ asymptotically?
- Imaginary $J/\psi$ strong decay amplitude?

Perspectives

- Data from SND and CMD2
- More data from BABAR ($\times 2$) and Belle (?)
- BESIII: ISR now, scan 2012-2013
- PANDA could explore FFs below threshold through $p\bar{p} \rightarrow \pi^0 l^+ l^-$