t-quark as a resonance

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Plans

- ✓ Introduction
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- ✓ Spectral representation of propagator – general
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- ✓ V-A theory
- ✓ Conclusions

Introduction

t-quark is a short-living particle because of the open channels with W on mass shell, so it may be considered as a resonance state. Similar to ordinary hadron resonances, the dressed t-quark propagator can be obtained as a result of Dyson summation of self-energy insertions or, equivalently, by solving the Dyson-Schwinger equation.

But for t-quark the vertex violates parity, so γ^5 takes part in this process and it generates nonstandard form of resonance – it differs from usual Breit-Wigner term.

This question was discussed earlier, see e.g. B.A.Kniehl and A.Sirlin. PRD77 (2008) 116012, where the dressed fermion propagator was obtained in analogy with boson one, i.e. without separation positive and negative energy poles.

Here we make the next step: we build the projectors onto the positive (negative) energy poles and study the corresponding resonance factors. It is possibly to do for arbitrary form of interaction, the particular case is V-A vertex of Standard Model.

To obtain Breit-Wigner-like formula in QFT we need to solve the Dyson-Schwinger equation for dressed propagator

$$G = G_0 + G_0 \Sigma G, \quad \text{or} \quad G^{-1} = G_0^{-1} - \Sigma \tag{1}$$

where G_0 and G are bare and dressed propagators and Σ is the self-energy. For bosons it gives:

$$G_0 = \frac{1}{m_0^2 - s - i\epsilon} \;\Rightarrow\; G = \frac{1}{m_0^2 - s - \Sigma(s)} \sim \frac{1}{m^2 - s - i\Gamma m}, \tag{2}$$

and if Σ has imaginary part, the dressed propagator G should be compared with relativistic Breit-Wigner formula.
For fermions all is similar

$$G_0 = \frac{1}{\hat{p} - m_0} \;\Rightarrow\; G = \frac{1}{\hat{p} - m_0 - \Sigma(p)}, \tag{3}$$

but to do this procedure more transparent, it's convenient to pass to off-shell projection operators

Off-shell projection operators looks like:

$$\Lambda^{\pm} = \frac{1}{2}\left(1 \pm \frac{\hat{p}}{W}\right), \tag{4}$$

where $W = \sqrt{p^2}$ is invariant mass or rest-frame energy.
In this basis the dressing looks like

$$G_0 = \frac{1}{\hat{p} - m_0} = \Lambda^+ \frac{1}{W - m_0} + \Lambda^- \frac{1}{-W - m_0} \quad \Rightarrow \tag{5}$$

$$\Rightarrow \quad G = \Lambda^+ \frac{1}{W - m_0 - \Sigma_1(W)} + \Lambda^- \frac{1}{-W - m_0 - \Sigma_2(W)}, \tag{6}$$

where the self-energy also can be decomposed in this basis

$$\Sigma(p) = A(p^2) + \hat{p}B(p^2) = \Lambda^+(A + WB) + \Lambda^-(A - WB) \equiv \Lambda^+\Sigma_1(W) + \Lambda^-\Sigma_2(W).$$

Standard Breit-Wigner formulas

After it the positive energy pole should be compared with Breit-Wigner formula

$$\frac{1}{W - m_0 - \Sigma^1(W)} \sim \frac{1}{W - m + i\Gamma/2}. \tag{7}$$

The above formulas correspond to parity conservation.

For parity violation the projection basis (4) should be supplemented by elements with γ^5, it is convenient to choose basis as:

$$\mathcal{P}_1 = \Lambda^+, \quad \mathcal{P}_2 = \Lambda^-, \quad \mathcal{P}_3 = \Lambda^+\gamma^5, \quad \mathcal{P}_4 = \Lambda^-\gamma^5. \tag{8}$$

Now the decomposition of self-energy and propagator has four terms:

$$S = \sum_{M=1}^{4} \mathcal{P}_M S_M, \tag{9}$$

where the coefficients S_M have the obvious symmetry properties:

$$S_2(W) = S_1(-W), \quad S_4(W) = S_3(-W) \tag{10}$$

and are calculated as

$$\begin{aligned} S_1 &= \frac{1}{2} Sp(\mathcal{P}_1 S), \qquad & S_2 &= \frac{1}{2} Sp(\mathcal{P}_2 S) \\ S_3 &= \frac{1}{2} Sp(\mathcal{P}_4 S), \qquad & S_4 &= \frac{1}{2} Sp(\mathcal{P}_3 S). \end{aligned} \tag{11}$$

Let's denote by $S(p)$ and $S_0(p)$ the dressed and bare inverse propagators. With using of decomposition (9), the Dyson-Schwinger equation (1) is reduced to set of equations for scalar coefficients

$$S_M = (S_0)_M - \Sigma_M\,, \qquad M = 1\dots4. \tag{12}$$

If to consider the self-energy Σ as a known value, we obtain the dressed propagator

$$G = \sum_{M=1}^{4} \mathcal{P}_M G_M\,, \tag{13}$$

where the coefficients G^M are

$$G_1 = \frac{S_2}{\Delta}, \quad G_2 = \frac{S_1}{\Delta}, \quad G_3 = -\frac{S_3}{\Delta}, \quad G_4 = -\frac{S_4}{\Delta}, \tag{14}$$

and $\Delta = S_1 S_2 - S_3 S_4$.

In spite of simple answer (13)(Kaloshin,Lomov), it is unconvinient because the positive and negative energy poles are not separated, compare with (6). So, we want to obtain analog of

$$G = \Lambda^{+} \frac{1}{W - m_0 - \Sigma_1(W)} + \Lambda^{-} \frac{1}{-W - m_0 - \Sigma_2(W)}, \tag{15}$$

for parity non-conservation case.

Spectral representation of propagator

To obtain the analog of (6) with parity violation, we will use the spectral representation of inverse propagator

$$S = \lambda_1 \, \Pi_1 + \lambda_2 \, \Pi_2, \tag{16}$$

where Π_i are projectors, satisfying the eigenstate problem

$$S \, \Pi = \lambda \, \Pi. \tag{17}$$

Let the dressed inverse propagator has the form:

$$S = \sum_{M=1}^{4} \mathcal{P}_M S_M,$$

with some coefficients S_M.
It's convenient to look for Π in form of decomposition (9). Easy to find that eigenvalues are roots of equation

$$\lambda^2 - \lambda(S_1 + S_2) + S_1 S_2 - S_3 S_4 = 0. \tag{18}$$

After some algebra one can find the projectors:

$$\Pi_1 = \frac{\lambda_2 - S_1}{\lambda_2 - \lambda_1}\left(\mathcal{P}_1 - \mathcal{P}_4\frac{S_4}{S_2 - \lambda_1}\right) + \frac{S_1 - \lambda_1}{\lambda_2 - \lambda_1}\left(\mathcal{P}_2 - \mathcal{P}_3\frac{S_3}{S_1 - \lambda_1}\right)$$

$$\Pi_2 = \frac{S_1 - \lambda_1}{\lambda_2 - \lambda_1}\left(\mathcal{P}_1 - \mathcal{P}_4\frac{S_4}{S_2 - \lambda_2}\right) + \frac{\lambda_2 - S_1}{\lambda_2 - \lambda_1}\left(\mathcal{P}_2 - \mathcal{P}_3\frac{S_3}{S_1 - \lambda_2}\right) \quad (19)$$

Their properties:

- ✓ $S(p)\,\Pi_i = \lambda_i\,\Pi_i$, where the eigenvalues λ_i are roots of equation (18).
- ✓ $\Pi_i^2 = \Pi_i$
- ✓ $\Pi_1\,\Pi_2 = \Pi_2\,\Pi_1 = 0$
- ✓ $\Pi_1 + \Pi_2 = 1$

Note that we rewrited $S(p)$ in equivalent form: using the explicite form of projectors (19) one can show that

$$\sum_{i=1}^{4}\mathcal{P}_i S_i \equiv \lambda_1\Pi_1 + \lambda_2\Pi_2. \quad (20)$$

Spectral representation of propagator

The dressed propagator $G(p)$ is obtained by reversing of (16):

$$G(p) = \frac{1}{\lambda_1}\,\Pi_1 + \frac{1}{\lambda_2}\,\Pi_2. \tag{21}$$

Let's look at the determinant $\Delta(W)$

$$\Delta(W) = S_1 S_2 - S_3 S_4 = (W - m_0 - \Sigma_1)(-W - m_0 - \Sigma_2) - \Sigma_3\Sigma_4, \tag{22}$$

where $\Sigma_i(W)$ are the self-energy components in basis (8). Bare propagator has poles at points $W = m_0$ and $W = -m_0$, the dressed one at $W = m$ and $W = -m$. But Δ is equal to product of eigenvalues

$$\Delta(W) = \lambda_1(W)\lambda_2(W), \tag{23}$$

so in the spectral representation of propagator (21) positive and negative energy poles contributions are separated from each other. The matrices (19) are projectors onto these poles.

Solving equation (18), we obtain eigenvalues at any form of self-energy:

$$\lambda_{1,2}(W) = -\left(m + \frac{\Sigma_1(W) + \Sigma_2(W)}{2}\right) \pm \sqrt{\left(W - \frac{\Sigma_1(W) - \Sigma_2(W)}{2}\right)^2 + \Sigma_3\Sigma_4} \tag{24}$$

Remark

In fact we used for our purposes the so called spectral representation of operator(see, e.g. textbook of A.Messia on quantum mechanics). In quantun-mechanical notations it has the form:

$$\hat{A} = \sum_i \lambda_i \; \Pi_i = \sum_i \lambda_i \; |i\rangle\langle i|,$$

where $|i\rangle$ are eigenvectors

$$\hat{A}|i\rangle = \lambda_i \; |i\rangle,$$

and $\Pi_i = |i\rangle\langle i|$ are corresponding projectors.

Decay width and self-energy

There is a known relation between decay width and imaginary part of self-energy, which does not depend on form of interaction. Consider for simplicity two-particle decay $N'(p) \to N(p_1)\pi(k_1)$

$$d\Gamma = \frac{1}{2M} \mid \mathcal{M} \mid^2 \frac{d^3 p_1}{2p_1^0 (2\pi)^3} \frac{d^3 k_1}{2k_1^0 (2\pi)^3} (2\pi)^4 \delta^4(p - p_1 - k_1). \tag{25}$$

Using the equality

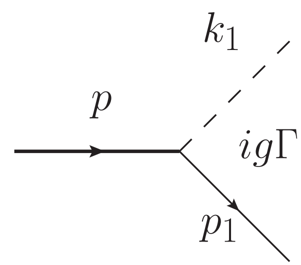
$$d^3 p_1 = d^4 p_1 \delta(p_1^2 - m^2) 2p_1^0 \theta(p_1^0), \tag{26}$$

one can rewrite width as four-dimentional integral

$$\Gamma = \frac{1}{2M} \int \frac{d^4 k_1}{(2\pi)^2} \mid \mathcal{M} \mid^2 \cdot \delta(k_1^2 - m_\pi^2) \theta(p_1^0) \cdot \delta((p_1 - k_1)^2 - m_N^2) \theta(p_1^0 - k_1^0). \tag{27}$$

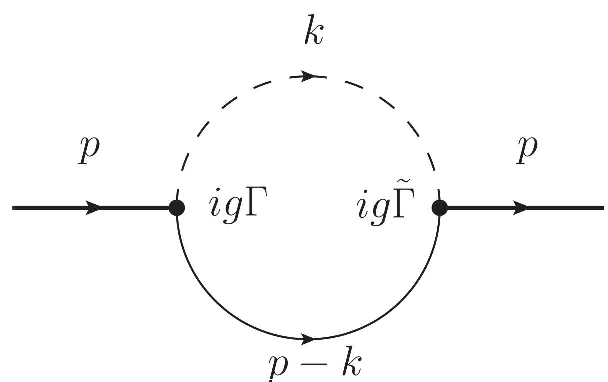
It looks like the discontinuty of loop, calculated according to Landau-Cutkosky rule. So we should write down the decay matrix element and corresponding self-energy.

Matrix element:



$$\mathcal{M} = g\bar{u}(p_1, s_1)\Gamma u(p, s) \quad \Rightarrow \quad \frac{1}{2}\sum_{s,s_1} \mid \mathcal{M} \mid^2 = \frac{g^2}{2} Sp\ (\hat{p} + m)\Gamma(\hat{p}_1 + m_N)\tilde{\Gamma}.$$

Self-energy:



$$\Sigma(p) = ig^2 \int \frac{d^4k}{(2\pi)^4}\ \Gamma \frac{1}{\hat{p} - \hat{k} - m_N} \tilde{\Gamma} \cdot \frac{1}{k^2 - m_\pi^2}$$

$$\Delta\Sigma(p) = -ig^2 \int \frac{d^4k}{(2\pi)^2}\ \tilde{\Gamma}(\hat{p} - \hat{k} + m_N)\Gamma \cdot \delta(k_1^2 - m_\pi^2)\theta(p_1^0) \cdot \delta((p_1 - k_1)^2 - m_N^2)\theta(p_1^0 - k_1^0$$

Decay width and self-energy

Compare

$$\frac{1}{2}\sum_{s,s_1} \mid \mathcal{M} \mid^2 = mg^2 Sp\ \frac{(\hat{p}+m)}{2m}\Gamma(\hat{p}_1 + m_N)\tilde{\Gamma}.$$

and rules of projection:

$$\Sigma = \sum_{M=1}^{4} \mathcal{P}_M \Sigma_M, \quad (28)$$

$$\Sigma_1 = \frac{1}{2} Sp(\mathcal{P}_1 \Sigma) = \frac{1}{2} Sp\ (\frac{\hat{p}+W}{2W}\Sigma).$$

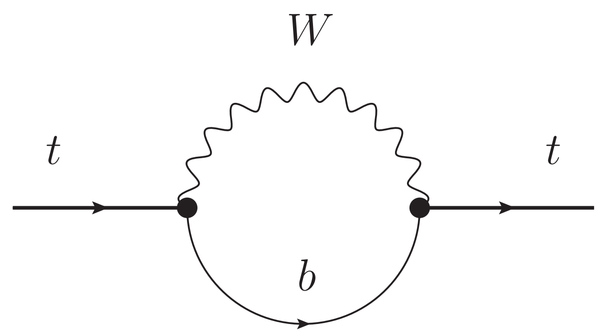
Appearence of $\frac{(\hat{p}+m)}{2m}$ in Γ tells that Γ is related with the first component Σ_1 of our decomposition.
As a result, we obtain the following relation between width and self-energy:

$$Im\ \Sigma_1(W=m) = -\frac{\Gamma}{2}. \quad (29)$$

Note that relation (29) is valid independently on parity violation.

V – A vertex

Consider the particular case of the above formulas, when vertex has $V - A$ structure. The main 1-loop contribution to self-energy in SM is:



$$\Sigma(p) = ig^2 \mid V_{tb} \mid^2 \int \frac{d^4k}{(2\pi)^4} \gamma^\mu (1-\gamma^5) \frac{1}{\hat{p} - \hat{k} - m_b} \gamma^\nu (1-\gamma^5) \frac{\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{m_W^2}\right)}{m_W^2 - k^2} =$$

$$= \hat{p}(1-\gamma^5)\Sigma_0(p^2).$$

Its decomposition in our basis (8):

$$\Sigma_1 = W\Sigma_0(W^2), \quad \Sigma_2 = -W\Sigma_0, \quad \Sigma_3 = -W\Sigma_0, \quad \Sigma_4 = W\Sigma_0.$$

General relation (29) now looks like:

$$Im\ \Sigma_0(W^2 = m^2) = -\frac{\Gamma}{2m} \tag{30}$$

Eigenvalues (24) take form

$$\lambda_{1,2}(W) = -m \pm W\sqrt{1 - 2\Sigma_0(W^2)}. \tag{31}$$

In analogy with OMS scheme let's subtract the real part of self-energy:

$$\lambda_{1,2}(W) = -m \pm W\sqrt{1 - 2(\Sigma_0(W^2) - Re\ \Sigma_0(m^2) - (Re\ \Sigma_0)'(m^2)(W^2 - m^2))}, \tag{32}$$

we obtain resonance factor:

$$\frac{1}{\lambda_1(W)} = \frac{1}{W\sqrt{1 + i\frac{\Gamma}{m}} - m} \approx \frac{1}{W - m + i\frac{W\Gamma}{2m}} \quad \text{at} \quad \frac{\Gamma}{m} << 1, \tag{33}$$

More about resonance factor

- Subtraction is made so to keep the property $\lambda_2(W) = \lambda_1(-W)$
- For t-quark $\Gamma/M = 2.0\ GeV/172.9\ GeV \sim 0.012$ really is a small parameter

Conclusions

- We studied in detail dressing of fermion propagator in case of parity non-conservation. It differs from naively expected Breit-Wigner form both in resonance factor and corresponding projector. But for $\Gamma/M \;<<\; 1$ the resonance factor returns to standard form in case of SM vertex (we are lucky?).
- Properties of projectors should be studied, but it is seen, that these projectors do not commutate with spin projectors $(1+\gamma^5\hat{s})/2$ and it can generate non-trivial spin properties.
- For t-quark, because of $\Gamma/M \;<<\; 1$, deviation from Breit-Wigner form is small for SM vertex. Presence, for instance, of right currents will change form of resonance factor. Note, that measurement of Γ_t at LHC is a difficult experimental problem.
- The above formulas may be applied for τ-lepton. In experiment the time exponent is obsewrved, not width. The nontrivial projector gives some corrections to matrix element, which should reflected into angular distributions and spin properties.
- This technics (spectral representation of propagator) may be used for mixing problem also.

Eigenvalues in SM

$$\lambda_{1,2}(W) = -m \pm W\sqrt{1 - 2\Sigma_0(W^2)}. \tag{34}$$

Their product:

$$\lambda_1(W)\lambda_2(W) = m^2 - W^2(1 - 2\Sigma_0(W^2)) = m^2 - W^2 + 2W^2\Sigma_0(W^2). \tag{35}$$

Small wonder: after all the boson-like denominator has been restored.

$$\mathcal{P}_1 = \Lambda^+, \quad \mathcal{P}_2 = \Lambda^-, \quad \mathcal{P}_3 = \Lambda^+\gamma^5, \quad \mathcal{P}_4 = \Lambda^-\gamma^5. \tag{36}$$

Multiplication properties

	$\mathcal{P}_1$	$\mathcal{P}_2$	$\mathcal{P}_3$	$\mathcal{P}_4$
$\mathcal{P}_1$	$\mathcal{P}_1$	0	$\mathcal{P}_3$	0
$\mathcal{P}_2$	0	$\mathcal{P}_2$	0	$\mathcal{P}_4$
$\mathcal{P}_3$	0	$\mathcal{P}_3$	0	$\mathcal{P}_1$
$\mathcal{P}_4$	$\mathcal{P}_4$	0	$\mathcal{P}_2$	0