Precision measurement of the positive muon lifetime by the MuLan collaboration

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The predictive power of the Standard Model depends on well-measured input parameters.

\[ \alpha^{-1} \]

<table>
<thead>
<tr>
<th>PDG-2010</th>
<th>0.00037 ppm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>137.035999084(51)</td>
</tr>
</tbody>
</table>

\[ M_Z \]

- 23 ppm
- 91.1876(21) GeV

\[ G_F \]

- 8.6 ppm
- 1.16639(1)x10^{-5} GeV^{-2}

* from muon lifetime

\[ \sigma_{\tau_{\mu}} = 18 \text{ ppm} \]

* before MuLan

http://lepewwg.web.cern.ch/LEPEWWG

PRL 100 (120801) 2008

\[ \chi^2/\text{DoF} = 2.2/3 \]

\[ \chi^2/\text{DoF} = 2.2/3 \]
Determination of $G_F$ from $\tau_\mu$

\[ \frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3} \left( 1 + \Delta q \right) \]

\[ \frac{\delta G_F}{G_F} = \frac{1}{2} \sqrt{ \left( \frac{\delta \tau_\mu}{\tau_\mu} \right)^2 + \left( 5 \frac{\delta m_\mu}{m_\mu} \right)^2 + \left( \frac{\delta (\Delta q)}{1 + \Delta q} \right)^2 } \]

mid 90 18 ppm 18 ppm 36 ppb 30 ppm

1999 9 ppm

Van Ritbergen and Stuart, 1999

PS+QCD+QED rad. corrections
**Motivation**

- $\tau_\mu$ can be used for the most precise determination of Fermi constant $G_F$.
- $\tau_\mu$ is needed for “reference” lifetime for precision muon capture experiments:
  - **MuCap:** $\mu^- + p$
  - **MuSun:** $\mu^- + d$

\[
\Delta \tau_{(\mu^+ - \mu^-)} \approx 0.16\%
\]

The singlet capture rate $\Lambda_s$ is used to determine $g_p$ and compare with theory:

\[
\Lambda_s = \frac{1}{\tau_{\mu^-}} - \frac{1}{\tau_{\mu^+}}
\]
πE3 Beamline at PSI

proton beam:
590 MeV
2.3 mA
The Kicker is used to create a pulsed beam

$$N_{in}(t) = \nu \tau \left( 1 - e^{-t/\tau\mu} \right)$$
The Kicker is used to create a pulsed beam

- **Kicker**
- **Separator**
- **Slit**
- **Beam**

**Diagram Details:**
- **Kicker:** +12.5 kV and -12.5 kV
- **Slit:** 5μs and 22μs
- **Al plates:** 12 cm x 75 cm
- **Graph:**
  - **Accumulation Period:**
  - **Measurement Period:**
  - **Equation:**
    \[ N(t) = N_0 e^{-t/\tau} + B \]
Stopping target

Target diameter: ~20 cm
Target thickness: ~0.5 mm
The target was opened once per day to monitor the beam.

RMS = 0.9 cm

RMS = 2.0 cm

Wire chamber
20μm W wires
1mm pitch
96 X-wires, 96 Y-wires
Efficiency > 95%
MuLan Ball

- scintillator
- 170 tiles
- diameter: ~80 cm
one tile is a pair of 3-mm thick BC-404 scintillators
base length ~ 15 cm

170 tiles
diameter: ~80 cm
MuLan Ball

pulse width ~6ns

2x170=340 detectors

170 tiles
diameter: ~80 cm

lightguide

MU
MuLan Ball

- Pulse width ~6ns
- 2x170 = 340 detectors
- Lightguide

Boston 450MHz WFD
- 4 channels/board
- Inner and outer tile as well as opposite tiles on one board
- 64 bit wide data bus to VME
- Total rate ~40 MB/s
- ~2x10^{12} pulses recorded
Data Analysis Flow

- Raw waveforms are fit with templates to find pulse amplitudes and times.
- Histogram times.
- Fit for $\tau_\mu$:
  \[ f(t) = N_0 e^{-t/\tau_\mu} + B \]

Blind analysis

- Exact clock frequency kept secret.
- Muon lifetime is reported with the value $R$ with units of ppm defined as:
  \[ \tau_\mu = \tau_{\text{secret}} (1 + R/10^6) \]
- Check consistency, study systematic errors.
What can go wrong?

Early-to-late changes, for instance:

**Instrumental issues**
- PMT gains
- Discriminator threshold walk
- Kicker voltage sag
- Pileup

**Physics issues**
- Spin polarization
- Non-flat background sources
Leading order pileup to a $\sim 5 \times 10^{-4}$ effect

$$f(t) = N_0 e^{-t/\tau_\mu} + N_2 e^{-2t/\tau_\mu} + N_3 e^{-3t/\tau_\mu} + \ldots + B$$
Pileup

Leading order pileup to a $\sim 5 \times 10^{-4}$ effect

- Statistically reconstruct pileup time distribution
- Fit corrected distribution

![Graph showing pileup time distributions](image)

Raw Spectrum

Pileup Corrected

Normal Time Distribution

Pileup Time Distribution

$\tau$

$\tau/2$
Pileup to sub-ppm requires higher-order terms

12 ns deadtime, pileup has a $5 \times 10^{-4}$ probability at our rates
Left uncorrected, lifetime wrong by 100's of ppm
Proof of procedure validated with detailed Monte Carlo simulation

D. M. Webber
The pileup corrections were tested with Monte-Carlo.

Monte-Carlo Simulation, $10^{12}$ events agrees with truth to < 0.2 ppm

$\chi^2 / \text{ndf} = 1.293 / 12$

Mean $= 100.1 \pm 0.0$

1.19 ppm statistical uncertainty
A slope exists due to a pileup undercorrection

Extrapolation to 0 deadtime is correct answer

Pileup Correction Uncertainty: 0.2 ppm
Precession of the muon spin distorts lifetime histogram

\[ \omega = g_\mu \frac{eB}{2m_\mu c} \]

\[ g_\mu \approx 2 \]

\[ \omega \sim 135 \text{ MHz/T} \]

\[ N(t) = N_0 e^{-t/\tau_\mu} \left[ 1 + A P_2 \cos(\omega t + \phi_0) \right] \]

This oscillation is easily detected

This oscillation is not easily detected and systematic errors may arise
μSR signal in opposite detectors

The sum cancels μSR effects

The difference accentuates μSR effects
MuLan strategy - reduce polarization!

- Dephasing
- Polarization destroying targets
Muons arrive randomly during 5μs accumulation period

Polarization of a muon ensemble

\[ \vec{P}(t) = \frac{1}{n} \sum_{i=1}^{n} \vec{s}(t) \]

\[ P(T_A) = \sqrt{\frac{\left( \cosh \left( \frac{T_A}{\tau_\mu} \right) - \cos(\omega T_A) \right) \csch^2 \left( \frac{T_A}{2\tau_\mu} \right)}{2(1 + \omega^2 \tau_\mu^2)}} \]

130 Gauss

4000 Gauss
Muon stopping targets

2006

Arnokrome-3 (AK-3) target
(~28% chromium, ~8% cobalt, ~64% iron)
0.4 T transverse field rotates muons with 
18 ns period

Muons precess by 0 to 350 revolutions
DEPHASED small ensemble avg.
polarization

2007

Crystalline quartz target
90% muonium formation
• 50% depolarization (Mu in singlet state)
• Fast precession of Mu in triplet state

10% “free” muons
• Noticeable precession
• Relaxation of longitudinal polarization
μSR magnet for Quartz target

Halbach array of permanent magnets to produce ~130 G field in the target plane
Two targets, two analysis strategies

**AK-3**
(strongly suppressed μSR)
Sum time histograms from all detectors and fit for $\tau_\mu$

**Quartz**
(noticeable μSR)
Incorporate μSR effects into the fit function. Fit each detector individually.
fit function

\[ f(t) = N_0 e^{-t/\tau_{\mu}} + B \]

\[ \tau_{\mu} = \tau_{\text{secret}}(1 + R/10^6) \]
Quartz target, fit individual detectors

**fit function**  
(most general form)

\[ f(t) = N_0 \left[ 1 + A\mathbf{\hat{P}}(t)\mathbf{\hat{D}} \right] e^{-t/\tau_{\mu}} + B \]

- decay asymmetry
- unit vector to detector
- ensemble polarization

**resolve \( \mathbf{P} \) into two components relative to B-field**

\[ f(t) = N_0 \left[ 1 + A(\mathbf{\hat{P}}_1(t) + \mathbf{\hat{P}}_2(t))\mathbf{\hat{D}} \right] e^{-t/\tau_{\mu}} + B \]

- transverse (precession+relaxation)
- longitudinal (relaxation)

\[ \mathbf{P}_1(t) = P_1 e^{-t/T_1} \quad P_1 \sim 0.0015, \quad T_1 \sim 28\mu s \]

\[ \mathbf{P}_2(t) = P_2 e^{-t/T_2} \cos(\omega t + \phi_0) \quad P_2 \sim 0.0025, \quad T_2 \sim 4\mu s \]

**practical realization** fit for muon disappearance time, then fit for \( \tau_{\mu} \)

\[ f(t) = N_0 \left[ 1 + A P_2 e^{-t/T_2} \cos(\omega t + \phi_0) \right] e^{-t/\tau_d} + B \]

\( \tau_d = \tau_{\mu} \left( 1 - A \frac{\tau_{\mu}}{T_1} \mathbf{\hat{P}}_1 \mathbf{\hat{D}} \right) \)

→ 170 values of \( \tau_d \)
fit results vs. detector position relative to B-field

\[ \chi^2 \]

\[ \chi^2 / \text{ndf} \]

\[ 163.9 / 169 \]

\[ p_0 \]

\[ 0.9997 \pm 0.003159 \]

\[ P_2 \]

\[ \chi^2 / \text{ndf} \]

\[ 223.3 / 125 \]

\[ p_0 \]

\[ 0.002462 \pm 1.7246 \times 10^{-5} \]

\[ \omega \]

\[ \chi^2 / \text{ndf} \]

\[ 67.03 / 81 \]

\[ p_0 \]

\[ 0.02557 \pm 1.4586 \times 10^{-5} \]

\[ \tau_d = \tau_\mu \left( 1 - A \frac{\tau_\mu}{T_1} \hat{P}_1 \hat{n}_D \right) \]
Consistency checks

Consistency against MANY special runs, where we varied target, magnet, ball
coincidences suppress background

\[ \frac{\delta N}{N} = 3 \times 10^{-4} \]
Correction for early-to-late gain variation

**Correction**

1) Gain vs. time variation is derived from the stability of the peak of the fit to pulse amplitude distribution
2) Extrapolate from MPV to threshold

**Consistency check**

Raise the threshold to amplify the effect

Gain correction is 0.5 ppm shift with 0.25 ppm uncertainty.
## Final Errors and Numbers (ppm units)

<table>
<thead>
<tr>
<th>Effect</th>
<th>2006</th>
<th>2007</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kicker extinction stability</td>
<td>0.20</td>
<td>0.07</td>
<td>Voltage measurements of plates</td>
</tr>
<tr>
<td>Residual polarization</td>
<td>0.10</td>
<td>0.07</td>
<td>Long relax; quartz spin cancelation</td>
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<tr>
<td>Upstream muon stops</td>
<td>0.10</td>
<td>0.20</td>
<td>Upper limit from measurements</td>
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<tr>
<td>Overall gain stability:</td>
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<td>MPV vs time in fill; includes:</td>
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<tr>
<td>Short time; after a pulse</td>
<td></td>
<td></td>
<td>MPVs in next fill &amp; laser studies</td>
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<tr>
<td>Long time; during full fill</td>
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<td>Different by PMT type</td>
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<td>Electronic ped fluctuation</td>
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<td></td>
<td>Bench-test supported</td>
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<tr>
<td>Unseen small pulses</td>
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<td></td>
<td>Uncorrected pileup effect → gain</td>
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<td>Timing stability</td>
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<td>Laser with external reference ctr.</td>
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<td>Pileup correction</td>
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<td>Extrapolation to zero ADT</td>
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<tr>
<td>Clock stability</td>
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<td></td>
<td>Calibration and measurement</td>
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<td><strong>Total Systematic</strong></td>
<td>0.42</td>
<td>0.42</td>
<td>Highly correlated for 2006/2007</td>
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<tr>
<td><strong>Total Statistical</strong></td>
<td>1.14</td>
<td>1.68</td>
<td></td>
</tr>
</tbody>
</table>

\[
\tau(R06) = 2\,196\,979.9 \pm 2.5 \pm 0.9 \text{ ps}
\]
\[
\tau(R07) = 2\,196\,981.2 \pm 3.7 \pm 0.9 \text{ ps}
\]
\[
\tau(\text{Combined}) = 2\,196\,980.3 \pm 2.2 \text{ ps (1.0 ppm)}
\]
\[
\Delta\tau(R07 - R06) = 1.3 \text{ ps}
\]

**New** \( G_F \)

\[
G_F(\text{MuLan}) = 1.166\,378\,8(7) \times 10^{-5} \text{ GeV}^{-2} \quad (0.6 \text{ ppm})
\]
Comparison of lifetime measurements

![Diagram showing comparison of lifetime measurements from different sources: Balandin - 1974, Giovanetti - 1984, Bardin - 1984, Chitwood - 2007, Barczyk - 2008, MuLan - R06, MuLan - R07. The diagram includes a highlighted area labeled "FAST."

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Thank you!