$g-2$: Status of the Standard Model prediction

Thomas Teubner

. Introduction

.. On behalf of Masashi Hayakawa:

- Status of the 10th order QED contributions to the leptonic $g-2$

... Recent developments in $(g-2)_\mu$: Hadronic Vacuum Polarisation contributions

- 2π channel: Inclusion of Radiative Return data
- Improvements in the region below 2 GeV
- pQCD or data for higher energies?

.... Full picture. Showdown. $\alpha(M_Z^2)$. Outlook

Thanks to my collaborators Kaoru Hagiwara, Ruofan Liao, Alan Martin and Daisuke Nomura.

$(g-2)_\mu$: Contributions

- $a_\mu = (g-2)_\mu/2 = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{had}} + a_\mu^{\text{New Physics?}}$
- QED: 4-loop predictions consolidated, 5-loop calculations ongoing, big surprises very unprobable for a_μ, error formidably small: $a_\mu^{\text{QED}} = 116584718.08(15) \cdot 10^{-11}$ ✓

  Kinoshita et al.

$\longrightarrow$ However, recent progress at 5-loop level very relevant for $\boldsymbol{a_e}$

$\hookrightarrow$ talk of Masashi Hayakawa

Current status of calculation of 10th-order QED contribution to lepton $g-2$

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Acknowledgment

I first thank to the organizers of this workshop and Dr. Teubner for special arrangement.

Subject

I overview the current status on the theoretical calculation of the 10th-order (5-loop) QED contribution to the lepton $g-2$

$$a_l(\mathrm{QED}) = \frac{\alpha}{\pi} a_{(1)} + \left(\frac{\alpha}{\pi}\right)^2 a_{l,(2)}(\mathrm{QED}) + \left(\frac{\alpha}{\pi}\right)^3 a_{l,(3)}(\mathrm{QED}) + \left(\frac{\alpha}{\pi}\right)^4 a_{l,(4)}(\mathrm{QED}) + \left(\frac{\alpha}{\pi}\right)^5 a_{l,(5)}(\mathrm{QED}) + \cdots ,$$

and present the *uncertainty* of the most accurate value of α derived using very preliminary result on full $a_{e,(5)}(\mathrm{QED})$ for the electron $g-2$.

Other works for 10th-order QED contributions

- estimate of logarithmic terms for muon $g-2$
  - A. L. Kataev, Phys. Rev. D **74**, 073011 (2006),
  - P. A. Baikov, K. G. Chetyrkin and C. Sturm, Nucl. Phys. Proc. Suppl. **183**, 8 (2008).
- analytic calculation of diagrams with bubbles of one-loop vacuum polarization
  - J. P. Aguilar, D. Greynat and E. De Rafael, Phys. Rev. D **77**, 093010 (2008).

Background

Comparison of values of α^{-1} provides a stringent test of our understanding on electromagnetic interaction based on quantum mechanics and field theory:

$$\alpha^{-1}(\mathrm{Rb06}) = 137\ 035\ 998\ 84\ (91)\ \ [6.7\mathrm{ppb}]\,,$$
$$\alpha^{-1}(\mathrm{Rb11}) = 137\ 035\ 999\ 037\ (91)\ \ [0.66\mathrm{ppb}]\,,$$
$$\alpha^{-1}(\mathrm{a_e08}) = 137\ 035\ 999\ 085\ (12)_8(37)_{10}(33)_{\mathrm{exp}}\ \ [0.37\mathrm{ppb}]\,,$$

where

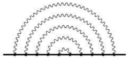
- $\alpha^{-1}(\mathrm{Rb11})$ is the latest result by R. Bourchendira, P.Clade, S.Guellati-Khelifa, F.Nez, and F.Biraben, Phys. Rev. Lett. **106**, 080801 (2011) (precise determination of $\hbar/m(\mathrm{Rb})$ using rubidium atom interferometer with Bloch oscillation),
- (*continued to the next slide*)

- $\alpha^{-1}(\mathrm{a_e}08)$ was obtained in T. Aoyama, M. Hayakawa, T. Kinoshita and M. Nio, Phys. Rev. Lett. **99**, 110406 (2007), which corrects the previous 8th-order calculation.
    - $(33)_{\mathrm{exp}}$ is the experimental error of $a_e \equiv (g_e - 2)/2$, B. C. Odom, D. Hanneke, B. D'Urso and G. Gabrielse, Phys. Rev. Lett. **97**, 030801 (2006),
    - $(12)_8$ is the theoretical uncertainty associated with the Monte Carlo integration of 8th-order QED correction, $a_{e,(4)}(\mathrm{QED})$,
    - $(37)_{10}$ is just the guess estimate on 10th-order QED correction, $a_{e,(5)}(\mathrm{QED})$.
    - Seriously, the reduction of the experimental error is scarified due to *lack of our knowledge on* $a_{e,(5)}(\mathrm{QED})$; $(37)_{10} \sim (33)_{\mathrm{exp}}$.

Strategy

- Full 10th-order QED calculation needs huge time, which exceeds one's life;
  - About 20 years for 8th order calculation
    $\Rightarrow$ More than 500 years needed for 10th order calculation...
  - Number of Feynman diagrams = 12,672,
  - Complicated ultra-violet and infrared divergent structures in each diagram
    $\Rightarrow$ Difficult to avoid mistakes in renormalization.
- We developed the automation scheme to generate FORTRAN-formatted programs for $g-2$ amplitude;
  Nucl. Phys. B **740**, 138 (2006) ; **796**, 184 (2008).
- (*continued to the next slide*)

- Genericity was discarded for implementation;
  - automation program 1 for the subset of quenched-type Feynman diagrams, whose number amounts to 6,354, about half of the total, 12,672,



  - automation program 2 for the subset 2,
  - automation program 3 for the subset 3,
  - ...

  Choice of such a strategy has made 10th-order project in progress quite efficiently.
- Numerical computation has been done using computational resources at RIKEN (*RSCC*, *RICC*) over these 6 years (*not 500 years*), in collaboration with
  - *Nio* (RIKEN),
  - *Asano, Watanabe, Aoyama* (Nagoya),
  - *Kinoshita* (Cornell).

Result

We have a *very preliminary result on full 10-th order QED contribution to* $g_e - 2$, which gives

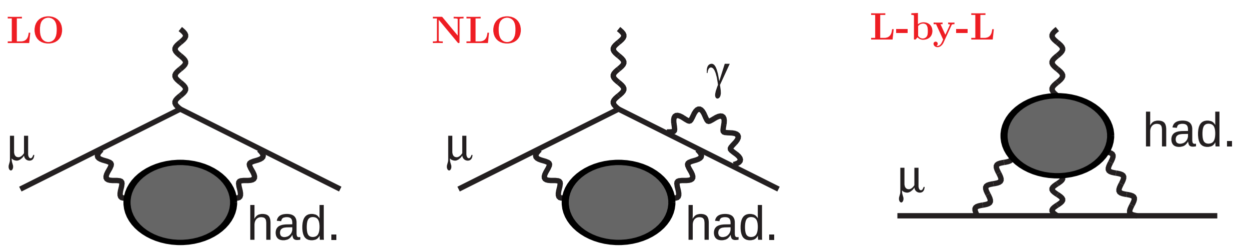
$$\alpha^{-1}(\mathrm{Rb06}) = 137\ 035\ 998\ 84\ (91)\ \ [6.7\mathrm{ppb}]\,,$$
$$\alpha^{-1}(\mathrm{Rb11}) = 137\ 035\ 999\ 037\ (91)\ \ [0.66\mathrm{ppb}]\,,$$
$$\alpha^{-1}(\mathrm{a_e}08) = 137\ 035\ 999\ 085\ (12)_8(37)_{10}(33)_{\mathrm{exp}}\ \ [0.37\mathrm{ppb}]\,,$$
$$\alpha^{-1}(\mathrm{a_e}11) = 137\ 035\ 999\ 1\cdot\cdot\ (09)_8(06)_{10}(33)_{\mathrm{exp}}\ [0.254\mathrm{ppb}]\,.$$

A few degists in $\alpha^{-1}(\mathrm{a_e}11)$ are dotted to avoid the value being quoted.

$(g-2)_\mu$: Contributions

- $a_\mu \;=\; (g-2)_\mu/2 \;=\; a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm had} \;+\; a_\mu^{\rm New\ Physics?}$
- QED: 4-loop predictions consolidated, 5-loop calculations ongoing, big surprises very unprobable for a_μ, error formidably small: $a_\mu^{\rm QED} = 116584718.08(15)\cdot 10^{-11}$ ✓
  Kinoshita et al.
- EW: consistent 2-loop predictions, accuracy fully sufficient: $a_\mu^{\rm EW} = (154\pm 2)\cdot 10^{-11}$ ✓
  Czarnecki et al., Knecht et al.
- Hadronic contributions: uncertainties completely dominate $\Delta a_\mu^{\rm SM}$ ×

$$a_\mu^{\rm had} \;=\; a_\mu^{\rm had,VP\ LO} \;+\; a_\mu^{\rm had,VP\ NLO} \;+\; a_\mu^{\rm had,Light-by-Light}$$



▶ Hadronic contributions from low γ virtualities not calculable with perturbative QCD

– Lattice simulations difficult: accuracy not (yet?!) competetive → K. Jansen

$(g-2)_\mu$: Contributions

- Hadronic Vacuum Polarisation from exp. $\sigma(e^+e^- \to \gamma^* \to hadrons(+\gamma))$ data

  [or from $\tau \to \nu_\tau + hadrons$ spectral functions; isospin breaking... $\longrightarrow$ talk of M. Benayoun]

  Use of dispersion integral (based on analyticity and unitarity):

$$a_\mu^{\rm had,VP\ LO} = \frac{1}{4\pi^3}\int_{m_\pi^2}^\infty {\rm d}s\, \sigma^0_{\rm had}(s) K(s)\,, \quad \text{with } K(s) = \frac{m_\mu^2}{3s}\cdot(0.4\ldots 1)$$

  $\longrightarrow$ Kernel $K \rightsquigarrow$ weighting towards smallest energies. $\sigma^0_{\rm had}$ the undressed cross section

  $\longrightarrow$ Similar approach with different kernel functions for NLO VP contributions $a_\mu^{\rm had,VP\ NLO}$

- Hadronic Light-by-Light: $\longrightarrow$ see also talk of A. Radzhabov

— No dispersion relation. *First Principles* calculations from lattice QCD are underway...

$\longrightarrow$ talk by K. Jansen

Also first results based on Dyson-Schwinger eqs. by C. Fischer et al.

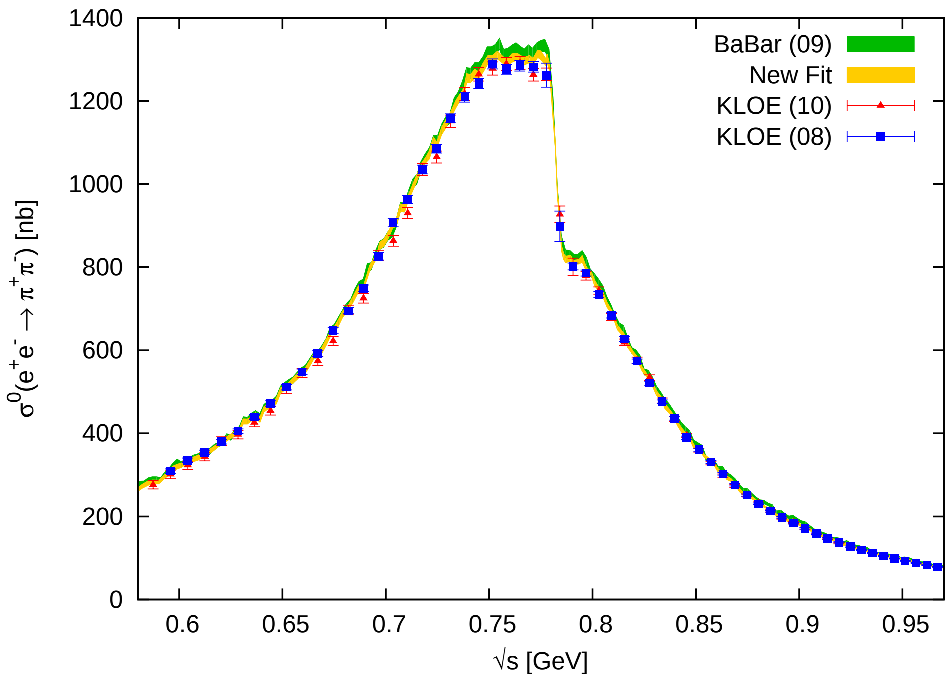
— 'Consensus' of different recent model calculations. HLMNT numbers below use compilation from J. Prades, E. de Rafael, A. Vainshtein: $\boldsymbol{a_\mu^{\rm L-by-L} = (10.5 \pm 2.6)\cdot 10^{-10}}$

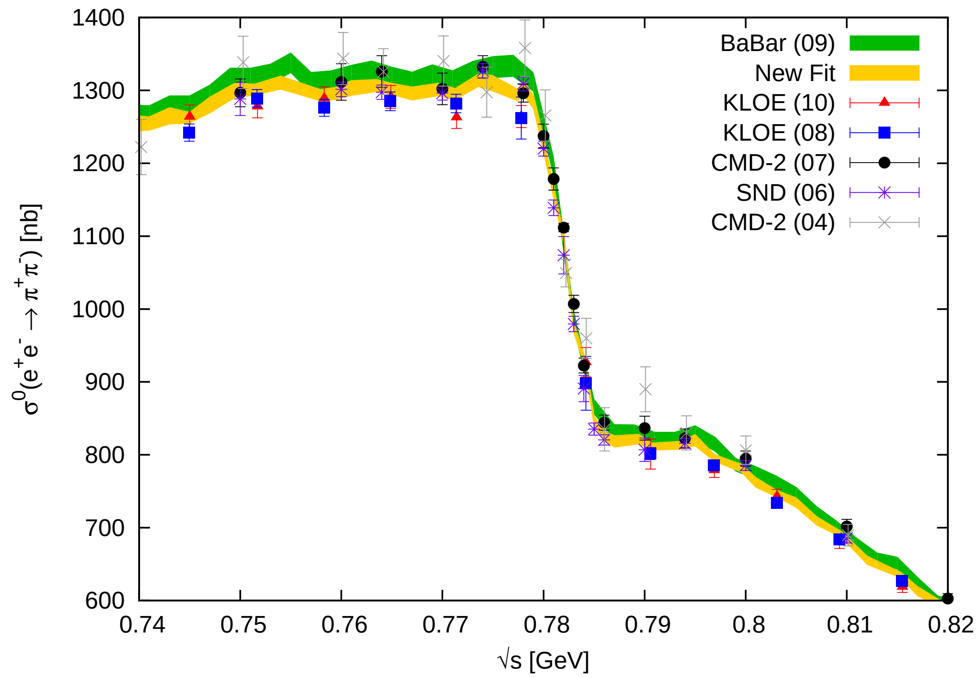
— Compatible result from F. Jegerlehner, A. Nyffeler: $\boldsymbol{a_\mu^{\rm L-by-L} = (11.6 \pm 4.0)\cdot 10^{-10}}$

Recent developments in $(g-2)_\mu$; Hadronic VP contributions

- For low energy $\sigma^0_{\rm had}(s)$, need to sum ~ 25 exclusive channels $[2\pi,\ 3\pi,\ KK,\ 4\pi,\ \ldots]$
- $\sqrt{s} \sim 1.4 - 2$ GeV: sum exclusive channels and/or use old inclusive data
- above ~ 2 GeV: inclusive data *or* use of perturbative QCD [+narrow resonances]

▶ The most important 2π channel ($> 70\%$) HLMNT '11 uses 879 data points

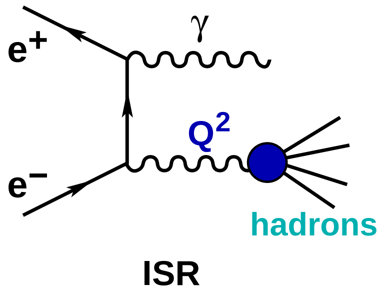




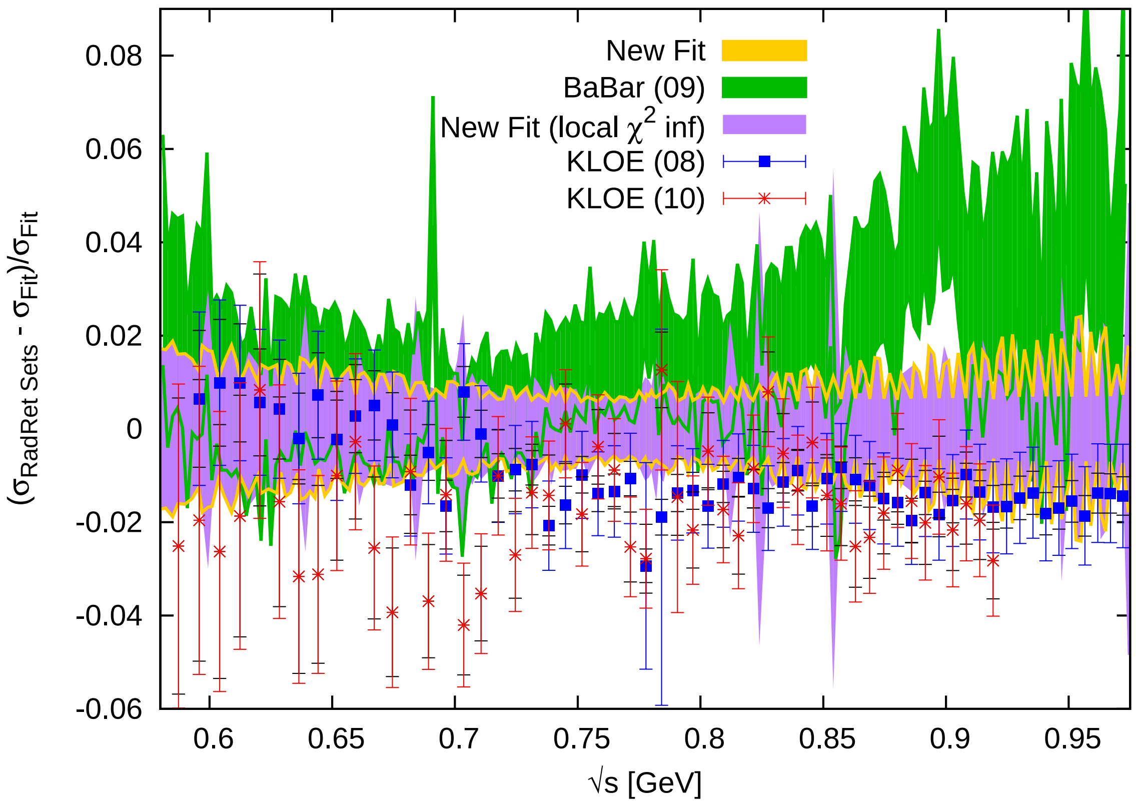
Overall, the data combination incl. 'Direct Scan' and 'Radiative Return' looks fine, but...

Radiative Return $\pi\pi(\gamma)$ data [KLOE 08/10 and BaBar 09] compared to combination of all

→ talks of P. Lukin, A. Hafner



Normalised difference of cross sections [HLMNT '11]



→ Radiative Return (at fixed e^+e^- energy) a powerful method, *complementary to direct energy scan*

⇝ Differences in shape and BaBar high at medium and higher energies

⇝ limited gain in accuracy due to 'tension'; pull-up (mainly from BaBar)

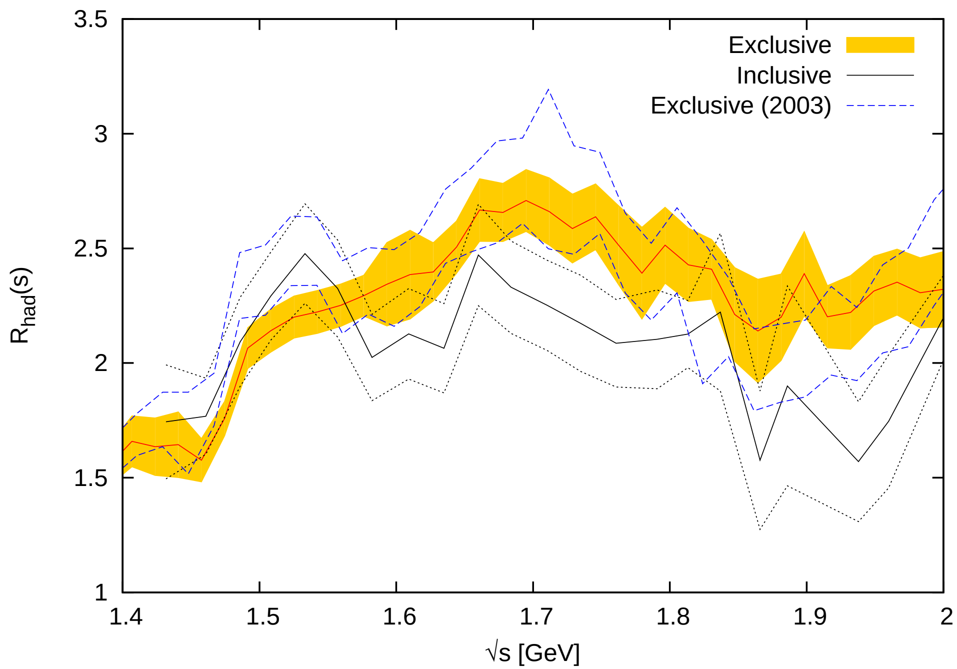
- Comb. of all data on same footing, before integration (purple band): still good $\chi^2_{\rm min}/{\rm d.o.f.} \sim 1.5$ of fit]
- $a_\mu^{2\pi}(0.32 - 2\,{\rm GeV}) = (\mathbf{504.2 \pm 3.0}) \cdot 10^{-10}$, $a_\mu^{2\pi,\mathbf{w/out Rad.\,Ret.}} = (\mathbf{498.7 \pm 3.3}) \cdot 10^{-10}$.
- Clarification/improvement with more, possibly even more precise data (from both scan and ISR)!?!

▶ Region below 2 GeV: many recent BaBar Rad. Ret. analyses

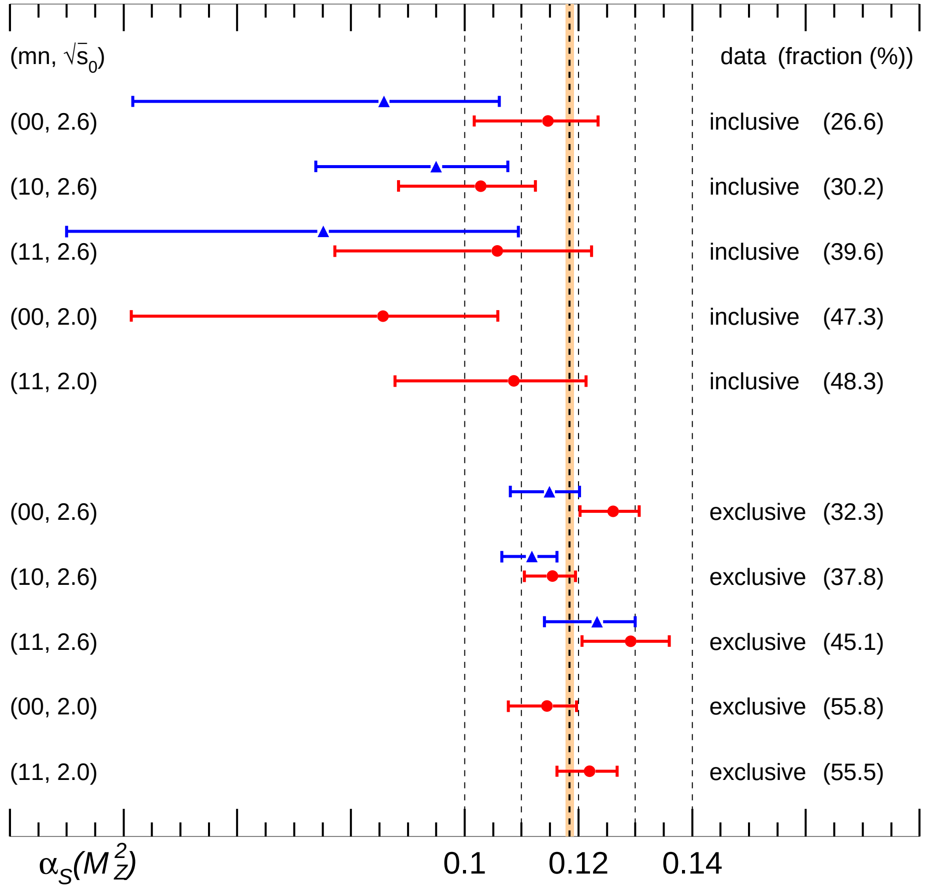
→ talk by A. Hafner

Data compilation $R_{\rm had} = \sigma_{\rm had}/[4\pi\alpha^2/(3s)]$:

blue: old excl. analysis, red/orange: new (2011)

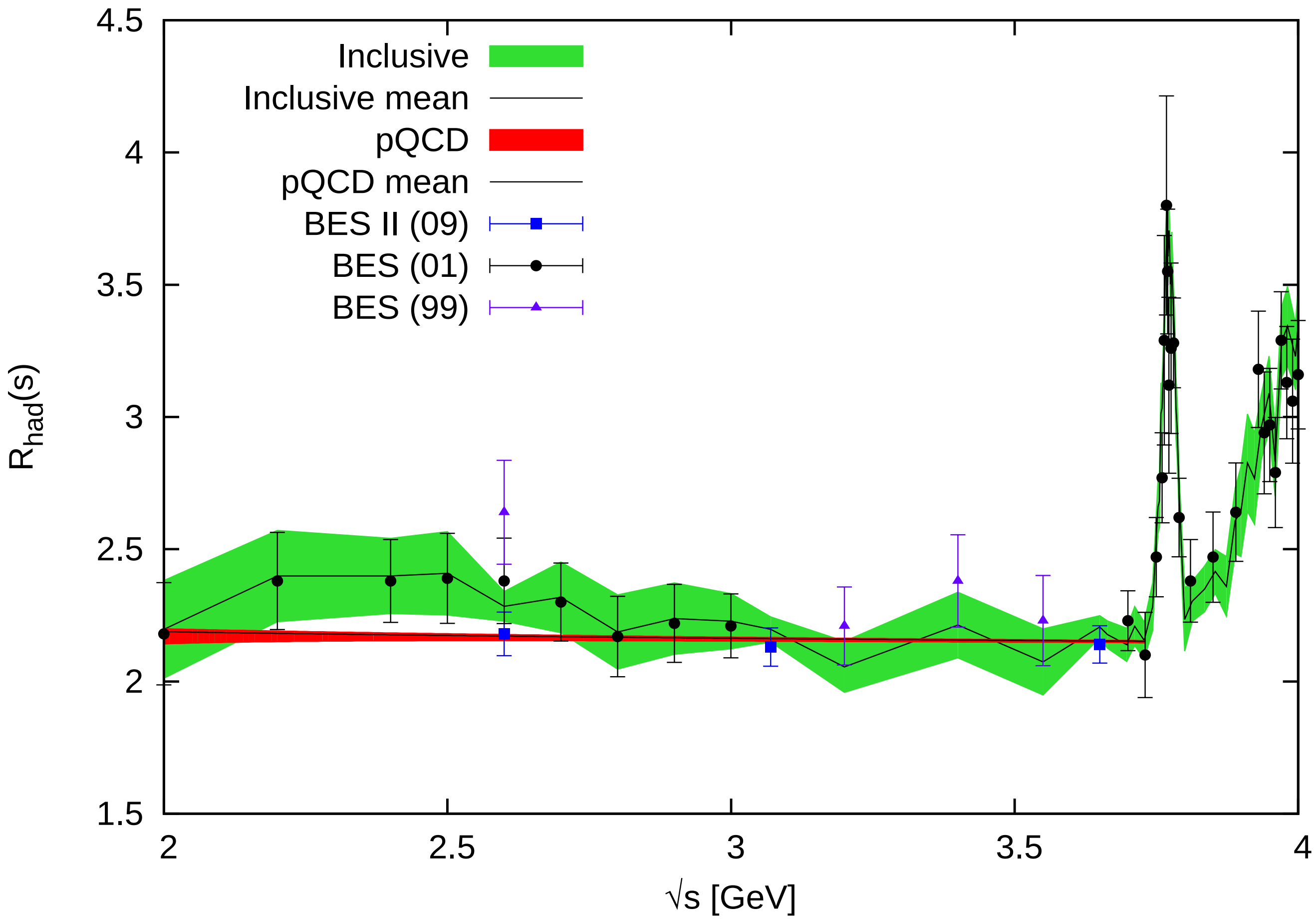


Sum-rules 'determining' α_s (2011):



- sum over exclusive channels improved
- shape similar, but normalisation (still) different from old inclusive data
- finite energy QCD sum rules ⇝ use of exclusive data now preferred over inlcusive
- Still rely on isospin relations for missing channels [sizeable error from $K\bar{K}\pi\pi$]
- For HLMNT '11: Use of more precise *sum over exclusive* (↪ shift up by $\sim\ +2.7\cdot 10^{-10}$ for a_μ)

▶ Perturbative QCD vs. inclusive data above 2 GeV (below the charm threshold)



- Latest BES data (blue markers) in perfect agreement with perturbative QCD; data slightly higher than pQCD for $\sqrt{s} > 2.6$ GeV
- HLMNT use pQCD for $2.6 < \sqrt{s} < 3.7$ GeV and with (larger) BES errors
  - would have small shift downwards ($\sim \mathbf{-1.4 \cdot 10^{-10}}$ for a_μ) if used from 2 GeV
  - Davier et al. use pQCD from 1.8 GeV

$a_\mu^{\rm SM}$: The full picture

[HLMNT '11: J. Phys. G **38** (2011) 085003]

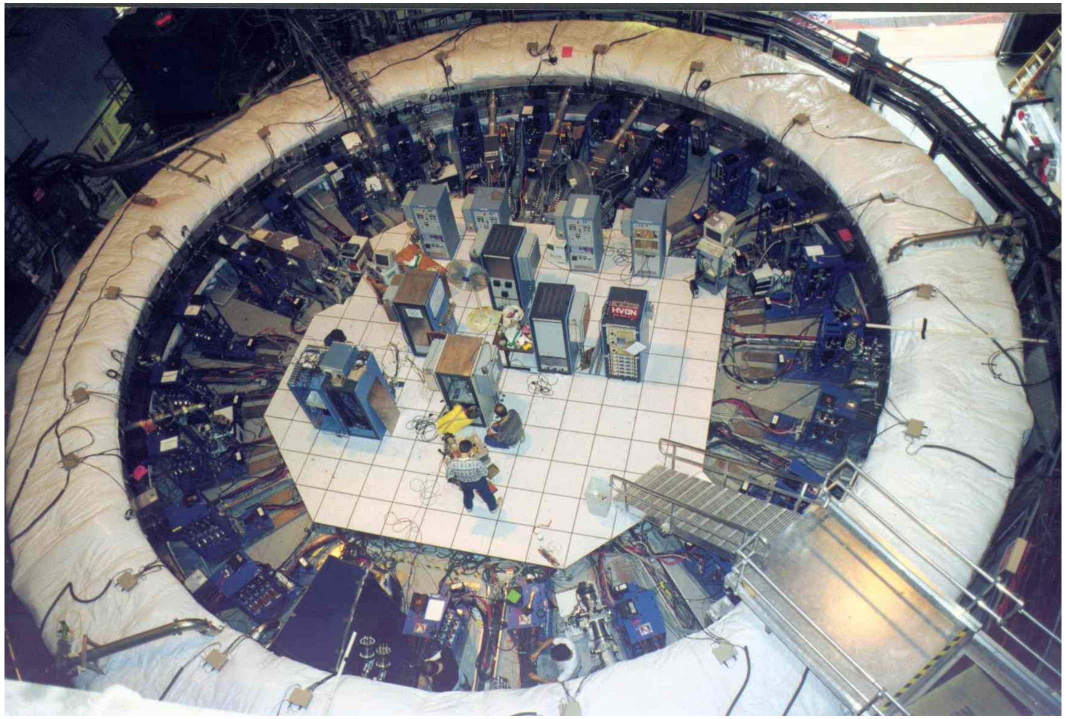
Source	contr. to $a_\mu \cdot 10^{11}$	remarks
QED	$116\,584\,718.08 \pm 0.15$	up to 5-loop (Kinoshita+Nio, Passera)
EW	154 ± 2	2-loop, Czarnecki+Marciano+Vainshtein
		(agrees very well with Knecht+Peris+Perrottet+deRafael)
LO hadr.	$\mathbf{6923 \pm 42}$	Davier *et al.* '10 (e^+e^-)
	$\mathbf{6908 \pm 47}$	F. Jegerlehner + R. Szafron '11 (e^+e^-)
	$\mathbf{6894 \pm 42 \pm 18}$	Hagiwara+Martin+Nomura+T '06
	$\mathbf{6949 \pm 37 \pm 21}$	HLMNT '11, as discussed. Combined error is 43
NLO hadr.	$\mathbf{-98.4 \pm 0.6 \pm 0.4}$	HLMNT, in agreem. with Krause '97, Alemany+D+H '98
L-by-L	105 ± 26	▶ Prades+deRafael+Vainshtein
agrees with	< 159 (95% CL)	upper bound from Erler+Toledo Sánchez from PHD
< Nov. 2001:	(-85 ± 25)	the 'famous' sign error, $2.6\,\sigma \to 1.6\,\sigma$
$\sum$	$\mathbf{116591828 \pm 49}$	HLMNT '11

The theory prediction of $g-2$ is now slightly more precise than the BNL measurement

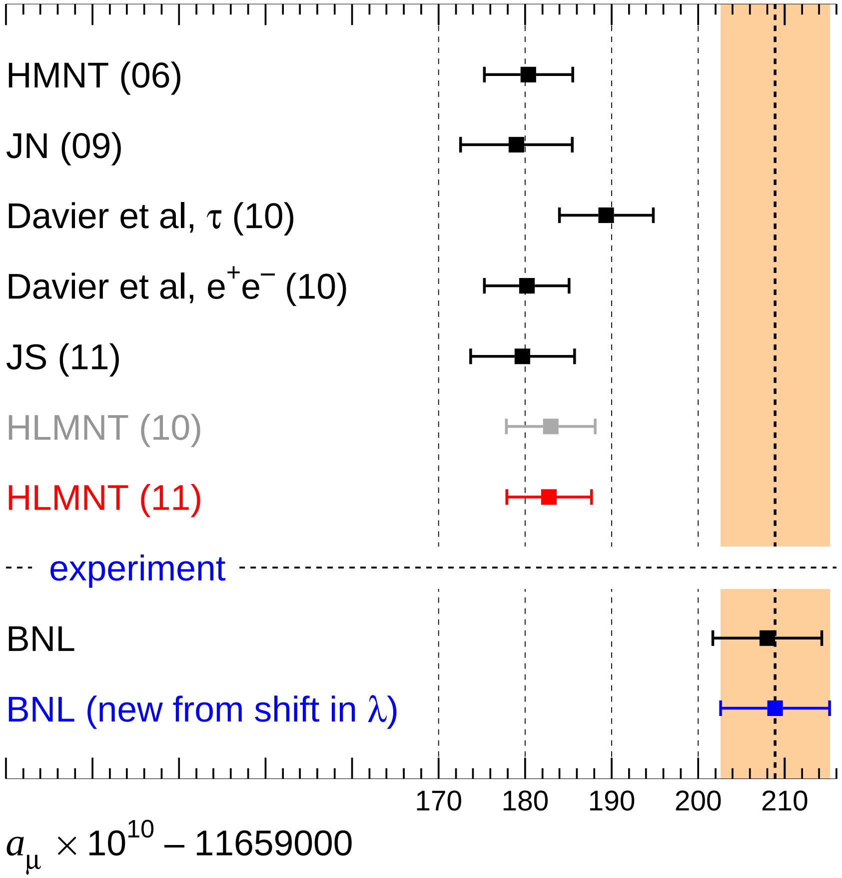
Showdown: SM vs BNL measurement

$\rightsquigarrow$ NP?! $\longrightarrow$ talk by D.Nomura

$$a_\mu^{\rm EXP} = 116\,592\,089(63) \cdot 10^{-11}\ (0.5\text{ppm})$$



Covered BNL storage ring (Pic. from the g–2 Collab.)



Various choices w.r.t. data, way to compile, τ (?!), L-by-L.

But: ALWAYS had and have found $a_\mu^{\rm SM} < a_\mu^{\rm EXP}$

► $a_\mu^{\rm EXP} - a_\mu^{\rm SM} = (26.1 \pm 8.0) \cdot 10^{-10}$

$\hookrightarrow$ **3.3 σ discrepancy** [using HLMNT '11]

— Davier et al. '10: **3.6 σ** $[e^+e^-]$

— Jegerlehner+Nyffeler '09: **3.2 σ**

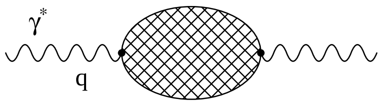
— J+Szafron '11: **3.3 σ** [incl. τ w. $\gamma - \rho$ mix.]

— HLMNT '09: was $4.0\,\sigma$ [w/out BaBar 09 2π]

$\longrightarrow$ talk by M. Benayoun:

$> 4\,\sigma$ incl. τ w. 'HLS', no ISR 2π data

The running QED coupling $\alpha(M_Z^2)$ ... and the Higgs mass

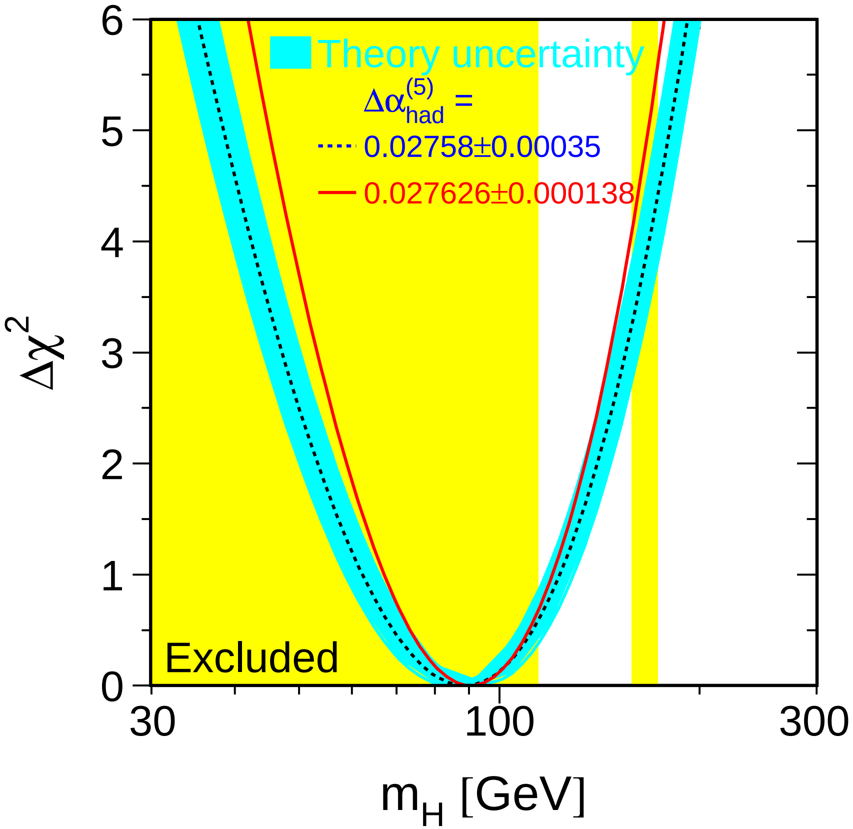


- Vacuum polarisation leads to the 'running' of α from $\alpha(q^2 = 0) = 1/137.035999084(51)$ to $\alpha(q^2 = M_Z^2) \sim 1/129$
- $\alpha(q^2) = \alpha/\left(1 - \Delta\alpha_{\rm lep}(q^2) - \Delta\alpha_{\rm had}(q^2)\right)$
- Again use of a dispersion relation: $\Delta\alpha^{(5)}_{\rm had}(q^2) = -\frac{\alpha q^2}{3\pi} P \int_{s_{\rm th}}^{\infty} \frac{R_{\rm had}(s)\, ds}{s(s-q^2)}$
- Hadronic uncertainties $\rightsquigarrow$ α the least well known EW param. of $\{G_\mu, M_Z, \alpha(M_Z^2)\}$!
- We find (HLMNT '11):

  $$\boldsymbol{\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.027626 \pm 0.000138}$$

  i.e. $\boldsymbol{\alpha(M_Z^2)^{-1} = 128.944 \pm 0.019}$
- HLMNT-routine for $\alpha(q^2)$ & $R_{\rm had}^{\rm data}$ available

Fit of the SM Higgs mass: LEP EWWG



Fit and Fig. thanks to M. Grünewald

$\hookrightarrow$ $\boldsymbol{M_H = 91^{+30}_{-23}}$ GeV

$[m_t = (173.3 \pm 1.1)$ GeV$]$

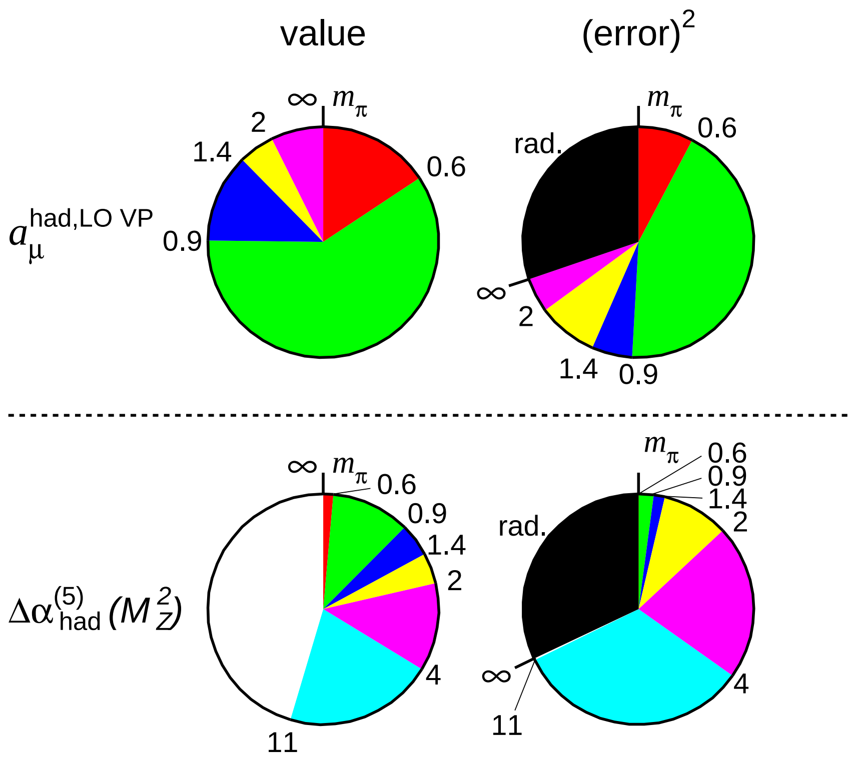
Outlook

- **Further improvements** Hadronic VP still the biggest error in $a_\mu^{\rm SM}$, soon L-by-L...

Prospects for further squeezing errors:

- More Rad. Ret. in progress at KLOE
- Great opportunity for KLOE-2, BELLE, Super $\tau - c$, in a few years SUPER-Bs, also strong case for DAFNE-HE
- Big improvement envisaged with CMD-3 and SND at VEPP2000
- Higher energies: BES-III at BEPCII in Beijing is on; KEDR at VEPP-4M

Pie diagrams for contr. to a_μ and $\alpha(M_Z)$ and their errors^2



- New $g-2$ experiments at Fermilab and J-PARC. $\longrightarrow$ talks by G. Venanzoni, N. Saito
- Will $a_\mu^{\rm SM}$ match the planned accuracy? $\rightsquigarrow$ L-by-L may become the limiting factor!

Conclusions

- $(g-2)_\mu$ strongly tests *all sectors* of the SM and constrains possible physics beyond.
- SM prediction consolidated in all sectors: Loops for QED + EW, many exp. data for R_{had} plus TH (incl. *Rad. Ret.*) for hadronic VP, low energy modelling for L-by-L.
- With the same data compilations as for $g-2$, also the hadronic contributions to $\Delta\alpha(q^2)$ have been determined; in turn $\boldsymbol{\alpha(M_Z^2)}$ has been improved considerably.
  $\hookrightarrow$ EW precision fits: $\boldsymbol{M_H = 91^{+30}_{-23}}$ GeV
- Interaction of TH + MC + EXP most important to achieve even higher precision.
  $\longrightarrow$ join the WG Radio Montecar Low meeting tomorrow
- low energy R_{had} is also a place to measure $\boldsymbol{\alpha_s}$ at a low scale.

▶ Discrepancy betw. SM pred. of $\boldsymbol{g-2}$ and BNL measurement persists at well $> \boldsymbol{3\,\sigma}$.

▶ More to come from all sides. Clear and strong case for continued *and* new experiments!

`The coming years will be exciting, and not only for the LHC`